# EFFECT OF ANISOTROPIC YIELD CRITERION ON SPRINGBACK IN PLANE STRAIN PURE BENDING

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The plastic anisotropy is one of various material representations that significantly affects springback simulations. In the present paper, the effect of plastic anisotropy of springback in plane strain pure bending is studied by means of an exact semi-analytical solution. To take into account anisotropy it is used the yield criterion and the constitutive equations for the orthotropic material with consideration of the crystal lattice constants and parameters of the crystallographic texture.

**Keywords:** anisotropy, crystallographic orientation, yield criterion, springback, bending, plane strain, elastic/plastic solution.

## Introduction

A comprehensive overview on springback that occurs following a sheet forming process when the forming loads are removed from the workpiece has been provided in [1]. It is emphasized in this overview that plastic anisotropy is one of various material representations that significantly affects springback simulations. In the present paper, the effect of plastic anisotropy of springback in plane strain pure bending is studied by means of an exact semi-analytical solution.

## 1. Material model considering crystallographic texture

The yield function of the orthotropic material considering the crystallographic texture has been proposed in [2]. Using this function and assuming the plane strain state ($\xi_{22} = \xi_{12} = \xi_{23} = 0$) the main equations of material model can be written as follows:

$$\sigma_{eq} = \frac{1}{\sqrt{2}} \sqrt{\frac{\eta_{31}^2}{\eta_{31} - \frac{1}{N}} \left(\sigma_{11} - \sigma_{33}\right)^2 + 4\left(\frac{5}{2} - \eta_{31}\right)\sigma_{31}^2}, \tag{1}$$

$$\xi_{eq} = \sqrt{2} \sqrt{\frac{\left(\eta_{12} + \eta_{23}\right)}{N\eta_{12}\eta_{23}\eta_{31}} \xi_{33}^2 + \frac{1}{4} \frac{\xi_{31}^2}{\frac{5}{2} - \eta_{31}}}, \tag{2}$$

$$\begin{aligned} \xi_{11} &= \frac{1}{2} \frac{\xi_{eq}}{\sigma_{eq}} \left[\eta_{12}\left(\sigma_{11} - \sigma_{22}\right) - \eta_{31}\left(\sigma_{33} - \sigma_{11}\right)\right], \\ \xi_{22} &= \frac{1}{2} \frac{\xi_{eq}}{\sigma_{eq}} \left[\eta_{23}\left(\sigma_{22} - \sigma_{33}\right) - \eta_{12}\left(\sigma_{11} - \sigma_{22}\right)\right], \\ \xi_{31} &= 2\frac{\xi_{eq}}{\sigma_{eq}} \left(\frac{5}{2} - \eta_{31}\right)\sigma_{31}, \end{aligned} \tag{3}$$

$$\sigma_{22} = \frac{\eta_{12}\sigma_{11} + \eta_{23}\sigma_{33}}{\eta_{12} + \eta_{23}}. \tag{4}$$

Here $\sigma_{eq}$ and $\xi_{eq}$ are the equivalent stress and strain rate, $\sigma_{ij}$ and $\varepsilon_{ij}$ are the physical components of the stress and strain rate in the Eulerian Cartesian system of coordinates. The generalized anisotropy factors $\eta_{ij}$ are defined as

$$\eta_{ij} = 1 - \frac{15(A'-1)}{3+2A'}\left(\Delta_i + \Delta_j - \Delta_k - \frac{1}{5}\right), \quad (5)$$

where $A'$ is the anisotropy factor of a crystal lattice, $\Delta_i$ are the parameters of the crystallographic texture. The anisotropy factor is defined through the compliance constants of crystal lattice $S'_{1111}$, $S'_{1122}$ and $S'_{2323}$

$$A' = \frac{S'_{1111} - S'_{1122}}{2S'_{2323}}. \quad (6)$$

For a certain crystallographic orientation $\{hkl\}\langle uvw\rangle$ the parameters of the crystallographic texture are defined as

$$\Delta_i = \frac{h_i^2 k_i^2 + k_i^2 l_i^2 + l_i^2 h_i^2}{\left(h_i^2 + k_i^2 + l_i^2\right)^2}, \quad (7)$$

where $h_i$, $k_i$, $l_i$ are Miller's indices, which determine the *i*-th direction in the crystal with respect to the principal axes of anisotropy.

## Elastic/plastic solution for plane strain pure bending

A general approach to analysis of plane strain pure bending has been proposed in [3]. The approach starts with the kinematics of the process, which is independent of constitutive laws. In particular, the following mapping between an Eulerian Cartesian coordinate system *xy* and a Lagrangian coordinate system $\zeta\eta$ has been introduced in [3]

$$\frac{x}{H} = \sqrt{\frac{\zeta}{a} + \frac{s}{a^2}}\cos(2a\eta) - \frac{\sqrt{s}}{a}, \qquad \frac{y}{H} = \sqrt{\frac{\zeta}{a} + \frac{s}{a^2}}\sin(2a\eta), \quad (8)$$

where *H* is the initial thickness of the sheet, *s* is an arbitrary function of *a*, *a* is a function of the time, *t*, and *a* = 0 at *t* = 0. It follows from (8) that $x = \zeta H$ and $y = \eta H$ at *a* = 0 if

$$s = 1/4 \text{ at } a = 0. \quad (9)$$

It can be verified by inspection by applying l'Hospital's rule to (8), with the use of (9), as $a \to 0$. Equations (8) and (9) describe a transformation of the rectangle defined at the initial instant, $a = 0$, by the equations $x = -H$, $x = 0$ and $y = \pm L$ (or, in the Lagrangian coordinates, by the equations $\zeta = -1$, $\zeta = 0$ and $\eta = \pm L/H$) into the shape determined by two circular arcs, *AD* and *CB*, and two straight lines, *AD* and *CB* (Fig. 1).

Equation (8) allows the principal strain rates to be calculated. These strain rates and constitutive equations are then substituted into the only non-trivial equilibrium equations in the Lagrangian coordinates. As a result, an equation for determining the function $s(a)$ is obtained. This equation should be solved numerically. Using this solution the state of stress and strain at the end of loading is calculated. It is then straightforward to find the shape of the sheet after unloading assuming that this process is purely elastic.

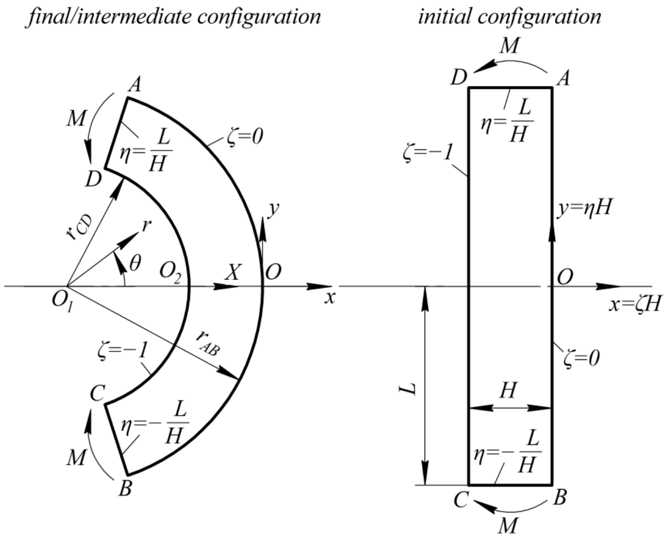


**Fig. 1.** Geometry of the process - notation

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