

# Modeling of a Fabry-Perot filter based on TiO<sub>2</sub> and air gap by using Eigenvectors and Eigenvalues approach

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**Abstract.** We used Eigenvectors and Eigenvalues method to obtain transmittance coefficients for each of an interference filter. The filter is based on titanium dioxide layer ( $n=2.26$ ) and air ( $n=1.0$ ). The transmission spectra of the FP filter in mid- IR region shows a characteristic peak with a maximum of 86 % at 4.70  $\mu\text{m}$  which is the fundamental absorption wavelength of CO which makes it possible to work as a gas sensor. The layers are horizontally stacked on one side which makes it possible to use air as a low index material. This gives an opportunity to deposit high index material in single deposition step which transforms the layer thickness into line thickness. As a result complex filter designs can be fabricated with simple and low production cost.

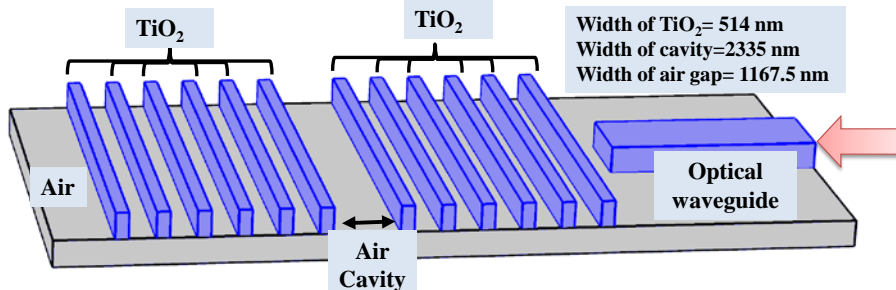
**Keywords:** Fabry-Perot filter, spectral filter, air gap, Eigenvectors, Eigenvalues.

## 1. Introduction

An optical filter is a device which is persistently used in the alteration of the spectral intensity distribution or the state of polarization of the electromagnetic radiation incident on it. The change in the spectral intensity distribution may or may not be governed by the wavelength. Filters can be based on numerous different physical phenomena such as absorption, refraction, interference, diffraction, scattering and polarization [1-5]. Optical filters were revolutionized ~60 years ago when Baumeister demonstrated the possibility to optimize optical filters by using a computer program [6]. Hereafter, a wide range of complex design procedures has been established that depend on sophisticated numerical methods and the use of computer [7]. Interference filters are fabricated by depositing several multilayer thin films on a transparent/partially transparent substrate. Both the multilayer and the substrate contribute to the overall performance of the filter [7].

In this work, we used Eigenvectors and Eigenvalues method to obtain a transmission function of a Fabry-Perot filter based on TiO<sub>2</sub> and air. The filter is designed in such a way that layer stacks are placed on the substrate next to each other on its one side. All the layers of a particular material are deposited at the same time. By using such method the layer thickness is converted into line thickness and is determined by patterning the filters using photolithography which results in the realization of a complex filter design with high volume and low individual component cost. The filter is designed at the central wavelength of 4.70  $\mu\text{m}$  which corresponds to the absorption line of CO in mid-IR [8]. As

CO highly absorbs at this wavelength, the performance of the matched reflectors is disturbed by the existence of this gas. The band-pass transmission deteriorates as a function of CO gas concentration. TiO<sub>2</sub> layer is deposited in a single fabrication step and the low index layer can be left as an air gap. TiO<sub>2</sub> is used as a high index material because of its magnificent properties [9, 10]. The filter is coupled from one side with the help of an optical waveguide. The 3-D design on the filter is shown in figure 1.



**Figure 1.** 3-D view of a designed filter consisting of several layers that can be fabricated in one step. The open-air assembly provides a platform for trace gas sensors.

**2. Eigenvectors and Eigenvalues method**

We consider the following form of Helmholtz equation and reduce the dependence of z-axis because of the large width of the filter:

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + k^2 \varepsilon U = 0, \tag{1}$$

Where  $\varepsilon$  is dielectric coefficient,  $k$  - is wave vector.

The solution value of the x bounds should be equal to zero, which means:

$$U(0, y) = U(a, y) = 0. \tag{2}$$

Where  $a$  - is layer height.

Equation will be solved like this:

$$U(x, y) = \sum_p c_p(y) \sin\left(\frac{\pi p x}{a}\right). \tag{3}$$

Eq (3) is solved to obtain the second order derivatives:

$$\frac{\partial^2 U}{\partial x^2} = \sum_p -\left(\frac{\pi p}{a}\right)^2 c_p(y) \sin\left(\frac{\pi p x}{a}\right) \tag{4}$$

$$\frac{\partial^2 U}{\partial y^2} = \sum_p \frac{\partial^2 c_p(y)}{\partial^2 y} \sin\left(\frac{\pi p x}{a}\right) \tag{5}$$

Use eq (4) and (5) in eq (1) to get:

$$\sum_p -\left(\frac{\pi p}{a}\right)^2 c_p(y) \sin\left(\frac{\pi p x}{a}\right) + \frac{\partial^2 c_p(y)}{\partial^2 y} \sin\left(\frac{\pi p x}{a}\right) + k^2 \varepsilon c_p(y) \sin\left(\frac{\pi p x}{a}\right) = 0, \tag{6}$$

By multiplying eq (6) by  $\sin\left(\frac{\pi n x}{a}\right)$  and integrate by x from 0 to a:

$$\int_0^a \sum_p \left[ -\left(\frac{\pi p}{a}\right)^2 c_p(y) \sin\left(\frac{\pi p x}{a}\right) + \frac{\partial^2 c_p(y)}{\partial^2 y} \sin\left(\frac{\pi p x}{a}\right) + k^2 \varepsilon c_p(y) \sin\left(\frac{\pi p x}{a}\right) \right] \sin\left(\frac{\pi p x}{a}\right) dx = 0,$$

After integration:

$$\sum_p -\left(\frac{\pi p}{a}\right)^2 c_p(y) \delta(p-n) + \frac{\partial^2 c_p(y)}{\partial^2 y} \delta(p-n) + k^2 \varepsilon c_p(y) \delta(p-n) = 0,$$

Finally eq (7) is obtained

$$\frac{\partial^2 c_n(y)}{\partial^2 y} + \left( k^2 \varepsilon - \left( \frac{\pi n}{a} \right)^2 \right) c_n(y) = 0 \quad (7)$$

Eq (7) can be represented in the alternative form:

$$\frac{\partial^2 C}{\partial y^2} = HC, \quad (8)$$

The solution has the following form:

$$C(t) = \sum_{n=-N}^N \left( c_n^+ e^{\sqrt{\lambda_n} y} + c_n^- e^{-\sqrt{\lambda_n} y} \right) \vec{U}_{\lambda_n} \quad (9)$$

$U_{\lambda}$  - the matrix of eigenvectors of  $H$ ,  $\lambda$  a matrix of eigenvalues of  $H$ . Now we can write (9) in the following form:

$$C(y) = U_{\lambda} \left\{ \exp\{\sqrt{\lambda} y\} c^+ + \exp\{-\sqrt{\lambda} y\} c^- \right\}$$

And the derivative form:

$$\frac{\partial C}{\partial y} = U_{\lambda} \left\{ \sqrt{\lambda} \exp\{\sqrt{\lambda} y\} c^+ - \sqrt{\lambda} \exp\{-\sqrt{\lambda} y\} c^- \right\}$$

Initial conditions for an incident light for first filter layer:

$$\begin{cases} C(0) = U_{\lambda} (c^+ + c^-) = f(0) \\ \frac{\partial C(0)}{\partial y} = U_{\lambda} (\sqrt{\lambda} c^+ - \sqrt{\lambda} c^-) = g(0) \end{cases} \quad (10)$$

Constants are derived as follows:

$$\begin{cases} c^+ = \left( E - \sqrt{\lambda}^{-1} \sqrt{\lambda} \right)^{-1} \left( U_{\lambda}^{-1} f(0) - \sqrt{\lambda}^{-1} U_{\lambda}^{-1} g(0) \right) \\ c^- = \left( E + \sqrt{\lambda}^{-1} \sqrt{\lambda} \right)^{-1} \left( U_{\lambda}^{-1} f(0) + \sqrt{\lambda}^{-1} U_{\lambda}^{-1} g(0) \right) \end{cases} \quad (11)$$

In our case:

$$\sqrt{\lambda} = -\sqrt{\lambda},$$

Finally, we represent the equations system (11) as:

$$\begin{cases} c^+ = \frac{1}{2} \left( U_{\lambda}^{-1} f(0) - \sqrt{\lambda}^{-1} U_{\lambda}^{-1} g(0) \right) \\ c^- = \frac{1}{2} \left( U_{\lambda}^{-1} f(0) + \sqrt{\lambda}^{-1} U_{\lambda}^{-1} g(0) \right) \end{cases} \quad (12)$$

Equation system (12) give us solution coefficient for the first layer. In order get coefficients for next layer we need to equate  $n$  and  $n+1$  solutions:

$$\begin{bmatrix} C_n(y) \\ \frac{\partial C_n}{\partial y} \end{bmatrix} = \begin{bmatrix} C_{n+1}(y) \\ \frac{\partial C_{n+1}}{\partial y} \end{bmatrix}$$

For  $n$  solution in point "y" the following equations are obtained:

$$\begin{cases} C_n(y) = U_{\lambda_n} \left\{ \exp\{\sqrt{\lambda_n} y\} c_n^+ + \exp\{-\sqrt{\lambda_n} y\} c_n^- \right\} \\ \frac{\partial C_n}{\partial t} = U_{\lambda_n} \left\{ \sqrt{\lambda_n} \exp\{\sqrt{\lambda_n} y\} c_n^+ - \sqrt{\lambda_n} \exp\{-\sqrt{\lambda_n} y\} c_n^- \right\} \end{cases} \quad (13)$$

The equations for a  $n+1$  solution in point "y" will be similar:

$$\begin{cases} C_{n+1}(y) = U_{\lambda_{n+1}} \left( \exp\{\sqrt{\Lambda_{n+1}}y\} c_{n+1}^+ + \exp\{\sqrt{\Lambda_n}y\} c_{n+1}^- \right) \\ \frac{\partial C_{n+1}}{\partial y} = U_{\lambda_{n+1}} \left( \sqrt{\Lambda_{n+1}} \exp\{\sqrt{\Lambda_{n+1}}y\} c_{n+1}^+ + \sqrt{\Lambda_{n+1}} \exp\{\sqrt{\Lambda_{n+1}}y\} c_{n+1}^- \right) \end{cases} \quad (14)$$

Equations systems (10) and (11) can be represented in a matrix form, for instance, the first system we get:

$$U_{\lambda_n} \begin{bmatrix} \exp\{\sqrt{\Lambda_n}y\} & \exp\{\sqrt{\Lambda_n}y\} \\ \sqrt{\Lambda_n} \exp\{\sqrt{\Lambda_n}y\} & \sqrt{\Lambda_n} \exp\{\sqrt{\Lambda_n}y\} \end{bmatrix} \begin{bmatrix} c_n^+ \\ c_n^- \end{bmatrix} = \begin{bmatrix} C_n(y) \\ \frac{\partial C_n}{\partial y} \end{bmatrix}$$

To simplify the equation, we substitute the first part of system matrix with  $[A_n]$  to get:

$$[A_n] \begin{bmatrix} c_n^+ \\ c_n^- \end{bmatrix} = \begin{bmatrix} C_n(y) \\ \frac{\partial C_n}{\partial y} \end{bmatrix}$$

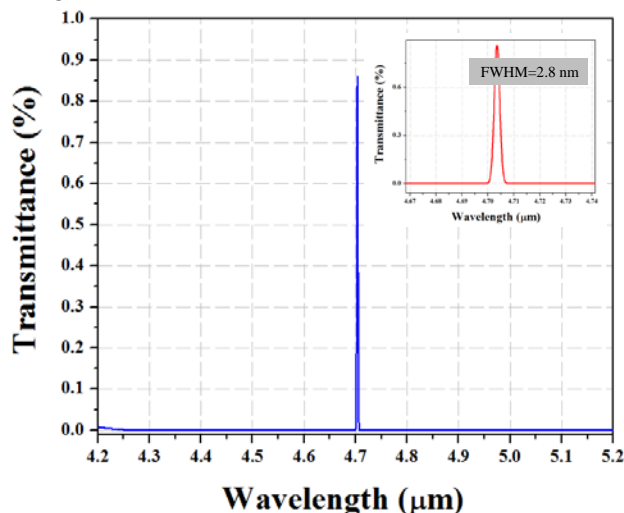
Similar to a  $n+1$  solution. Finally, we get:

$$[A_n] \begin{bmatrix} c_n^+ \\ c_n^- \end{bmatrix} = [A_{n+1}] \begin{bmatrix} c_{n+1}^+ \\ c_{n+1}^- \end{bmatrix}$$

Then we obtain the equation, which defines the transmittance and reflectance coefficients:

$$[A_{n+1}]^{-1} [A_n] \begin{bmatrix} c_n^+ \\ c_n^- \end{bmatrix} = \begin{bmatrix} c_{n+1}^+ \\ c_{n+1}^- \end{bmatrix} \quad (15)$$

The transmission spectrum of a FP filter with 41 layers of  $\text{TiO}_2/\text{air}$  is obtained by solving eq (15) for each layer as shown in figure 2.



**Figure 2.** The transmission spectrum of 41 layers  $\text{TiO}_2/\text{air}$  FP filter modeled at 4.7  $\mu\text{m}$  absorption line of CO.

### 3. Conclusion

In this work, we used Eigenvectors and Eigenvalues approach to design a FP filter by using air as a low index material. The transmission spectra of the FP filter in mid- IR region shows a characteristic peak with a maximum of 86 % at 4.70  $\mu\text{m}$  which is the fundamental absorption wavelength of CO so it has a potential to work as a gas sensor. This filter construction allows various characteristic design features containing a large number of layers and a high refractive index contrast between the high and low index materials.

#### 4. Acknowledgement

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