Modeling of entanglement behavior of two dipole-coupled superconducting qu interacting with quantum fields in lossless cavities

E.K. Bashkirov

Samara National Research University, 443086, 34 Moskovskoye shosse, Samara, Russia

Abstract

We have investigated the entanglement dynamics between two initially entangled superconducting qubits interacting with vacuum fields of one or two independent coplanar 1D resonators. Three types of two-qubits generalized Jaynes-Cummings models with different atom-field coupling constants and detunings and direct dipole-dipole interaction has been considered. Using the dressed-states technique we have derived the exact solutions for models under consideration. The computer modeling of the time dependence of qubit-qubit entanglement (negativity) has been carried out for different strength of the dipole-dipole interaction. Results show that dipole-dipole interaction may be used for entanglement control.

Keywords: superconducting qubits; microwave cavities; entanglement; detuning; dipole-dipole interaction

1. Introduction

The superconducting Josephson junctions are good candidates for the construction of quantum qubits for a quantum computer [1]. Quantum computers are devices that store information on quantum variables such as spins, photons, and atoms, and that process that information by making those variables interact in a way that preserves quantum coherence . To perform a quantum computation, one must be able to prepare qubits in a desired initial state, coherently manipulate superpositions of a qubits two states, couple qubits together, measure their state, and keep them relatively free from interactions that induce noise and decoherence [2]. Qubits have been physically implemented in a variety of systems, including cavity quantum electrodynamics, superconducting qubits, atoms and ions in traps, quantum dots, spins and hybrid systems [4]. The supercoducting Josephson circuits is attractive because the low dissipation inherent to superconductors make possible, in principle, long coherence times. In addition, because complex superconducting circuits can be microfabricated using integrated-circuit processing techniques, scaling to a large number of qubits should be relatively straightforward. The connection between qubits can be arranged through their interaction with fields of microwave coplanar resonators. Basic protocols of quantum physics calculations are based on the use of entangled states [1]. Therefore, great efforts have been made to investigate entanglement characterization, entanglement control, and entanglement production in different systems. It is well known that the JaynesCummings model (JCM) is the simplest possible physical model that describes the interaction of a natural or artificial two-level atom (qubit) with a single-mode cavity [3], and has been used to understand a wide variety of phenomena in quantum optics and condensed matter systems, such as superconducting circuits in microwave cavity [4]. In order to explore a wider range of phenomena caused by the interaction of the qubits with the quantum fields in resonators the numerous generalizations of the JCM have been investigated in recent years (see references in [5]-[7]). Yönac et al. [8, 9] have proposed the so-called double JCM (DJCM), consisting of two two-level atoms and two resonator modes, provided that each atom interacts only with one field of the resonator, and investigated the pairwise entanglement dynamics of this model. Recently, the DJCM have been extensively investigated [10]-[20].

The direct dipole-dipole interaction between the qubits is the natural mechanism of entanglement producing and controlling. It's very important that the effective dipole-dipole interaction for superconducting Josephson qubits may be much greater than the coupling between the qubit and cavity field [21]-[23]. The numerous references to the theoretical papers devoted to investigation of entanglement in two-qubit systems taking into account the dipole-dipole interaction are cited in our works [24]-[29]. In this paper, we considered a non-resonant double JCM taking into account the direct dipole-dipole interaction between qubits. We investigated the entanglement between qubits, and discussed the dependence of the entanglement on the parameters of the considered system, such as different intensity of dipole interaction, coupling constants and the detuning between the atomic transition frequency and the cavity field frequencies.

2. Double Jaynes-Cummings model

We consider two identical superconducting qubits labeled A and B, and two cavity modes of coplanar resonators labeled a and b. Qubit A not-resonantly interacts with a single-mode cavity field a, and qubit B not-resonantly interacts with a single-mode cavity field b. Due to the randomness of the qubits positions in the cavity, it is very difficult to control the couplings between different atom-cavity systems to be the same. Therefore the coupling constants between the atoms and cavities are assumed to be unequal. For superconducting qubits interacting with microwave coplanar resonators or *LC* superconducting circuits the intensity of effective dipole-dipole interaction can be compared with the atom-cavity coupling constant. In this case the dipole-dipole interaction should be included in the model Hamiltonian. Therefore in a frame rotating with the qubit frequency ω_0, the

Hamiltonian for the system under rotating wave approximation can be written as

$$H = \hbar\delta_a a^+ a + \hbar\delta_b b^+ b + \hbar g_a(\sigma_A^+ a + a^+\sigma_A^-) + g_b(\sigma_B^+ b + b^+\sigma_B^-) + \hbar J(\sigma_A^+\sigma_B^- + \sigma_A^-\sigma_B^+), \tag{1}$$

where $(1/2)\sigma_i^z$ is the inversion operator for the ith qubit ($i = A, B$), $\sigma_i^+ = |+\rangle_{ii}\langle -|$, and $\sigma_i^- = |-\rangle_{ii}\langle +|$ are the transition operators between the excited $|+\rangle_i$ and the ground $|-\rangle_i$ states in the ith qubit, a^+ and a are the creation and the annihilation operators of photons of the cavity mode a, b^+ and b are the creation and the annihilation operators of photons of the cavity mode b, $\hbar\omega_0$ is the superconducting gap energy, $g_a \equiv g$ is the coupling constant between qubit A and the cavity field a and g_b is the coupling constant between qubit A and the cavity field a, δ_a and δ_b are the detunigs for mode a and b and J is the coupling constant of the dipole interaction between the qubits A and B. The two-qubit wave function can be expressed as a combination of state vectors of the form $|v_1, v_2\rangle = |v_1\rangle|v_2\rangle$, where $v_1, v_2 = +, -$.

Firstly we take two qubits initially in the Bell-like pure state of the following form

$$|\Psi(0)\rangle_A = \cos\theta|+,-\rangle + \sin\theta|-,+\rangle$$

and the cavity fields initially in vacuum state $|0,0\rangle$. Here, $\cos\theta$ and $\sin\theta$ are the superposition coefficients. We take into account that optimal temperature at which the superconducting qubits are used for quantum computing is mK. For such temperature the influence of thermal photons of the microwave cavity field on the dynamics of qubits can be neglected.

Then the full initial state for considered model is

$$|\Psi(0)\rangle = (\cos\theta|+,-\rangle + \sin\theta|-,+\rangle) \otimes |0,0\rangle, \tag{2}$$

where $0 \le \theta \le \pi$.

The evolution of the system under consideration is confined in the subspace

$$|-,-,0,1\rangle, |-,-,1,0\rangle, |-,+,0,0\rangle, |+,-,0,0\rangle.$$

To obtain the time-dependent wave function of considered model one can use the so-called dressed states or eigenvectors of the Hamiltonian (1). We have obtained these for general case when parameters of the Hamiltonian (1) take the arbitrary values. But the general expressions for eigenvectors are too cumbersome to display here. Therefore, we present below the eigenvectors and eigenvalues of the Hamiltonian (1) for special case when $\delta_a = -\delta_b = \delta$ and $g_a = g_b = g$.

In this case the eigenvectors of the Hamiltonian (1) can be written as

$$|\Phi_i\rangle = \xi_i(X_{i1}|-,-,0,1\rangle + X_{i2}|-,-,1,0\rangle + X_{i3}|-,+,0,0\rangle + X_{i4}|+,-,0,0\rangle) \quad (i = 1,2,3,4), \tag{3}$$

where

$$\xi_i = 1/\sqrt{|X_{i1}|^2 + |X_{i2}|^2 + |X_{i3}|^2 + |X_{i4}|^2}$$

and

$$X_{11} = \frac{2\alpha}{\alpha^2 + \Delta^2 - B + \sqrt{2}\Delta\sqrt{A-B}}, \quad X_{12} = \frac{\sqrt{2}}{\Delta\sqrt{2} - \sqrt{A-B}}, \quad X_{13} = \frac{-\alpha^2 - \Delta^2 + B + \sqrt{2}\Delta\sqrt{A-B}}{\alpha\left(-2\Delta + \sqrt{2}\sqrt{A-B}\right)}, \quad X_{14} = 1;$$

$$X_{21} = \frac{2\alpha}{\alpha^2 + \Delta^2 - B - \sqrt{2}\Delta\sqrt{A-B}}, \quad X_{22} = \frac{\sqrt{2}}{\Delta\sqrt{2} + \sqrt{A-B}}, \quad X_{23} = \frac{\alpha^2 + \Delta^2 - B + \sqrt{2}\Delta\sqrt{A-B}}{\alpha\left(2\Delta + \sqrt{2}\sqrt{A-B}\right)}, \quad X_{24} = 1,$$

$$X_{31} = \frac{2\alpha}{\alpha^2 + \Delta^2 + B + \sqrt{2}\Delta\sqrt{A+B}}, \quad X_{32} = \frac{\sqrt{2}}{\Delta\sqrt{2} - \sqrt{A+B}}, \quad X_{33} = \frac{\alpha^2 + \Delta^2 + B - \sqrt{2}\Delta\sqrt{A+B}}{2\alpha\Delta - \sqrt{2}\alpha\sqrt{A+B}}, \quad X_{34} = 1,$$

$$X_{41} = \frac{2\alpha}{\alpha^2 + \Delta^2 + B - \sqrt{2}\Delta\sqrt{A+B}}, \quad X_{42} = \frac{\sqrt{2}}{\Delta\sqrt{2} + \sqrt{A+B}}, \quad X_{43} = \frac{\alpha^2 + \Delta^2 + B + \sqrt{2}\Delta\sqrt{A+B}}{\alpha\left(2\Delta + \sqrt{2}\sqrt{A+B}\right)}, \quad X_{44} = 1,$$

where $\Delta = \delta/\gamma$, $\alpha = J/\gamma$ and $A = 2 + \alpha^2 + \Delta^2$, $B = \sqrt{\alpha^4 + 4\Delta^2 + \Delta^4 - 2\alpha^2(-2 + \Delta^2)}$.

The corresponding eigenvalues are

$$E_1 = -\hbar\sqrt{A-B}/\sqrt{2}, \quad E_2 = \hbar\sqrt{A-B}/\sqrt{2}, \quad E_3 = -\hbar\sqrt{A+B}/\sqrt{2}, \quad E_4 = \hbar\sqrt{A+B}/\sqrt{2}.$$

For entanglement modeling we can obtain the time dependent wave function

$$|\Phi(t)\rangle = e^{-\imath Ht/\hbar}|\Phi(0)\rangle. \tag{4}$$

Using the eigenvalues and eigenvectors of Hamiltonian (1) and the initial state (2) we can derive from (4)

$$|\Psi(t)\rangle = C_1^{(1)}(t)|-,-,0,1\rangle + C_2^{(1)}(t)|-,-,1,0\rangle + C_3^{(1)}(t)|-,+,0,0\rangle + C_4^{(1)}(t)|+,+,n\rangle, \tag{5}$$

where

$$C_1^{(1)} = \cos\theta Z_{11} + \sin\theta Z_{12}, \quad C_2^{(1)} = \cos\theta Z_{21} + \sin\theta Z_{22},$$

$$C_3^{(1)} = \cos\theta Z_{31} + \sin\theta Z_{32}, \quad C_4^{(1)} = \cos\theta Z_{41} + \sin\theta Z_{42},$$

and

$$Z_{11} = e^{-\imath E_1 t/\hbar}\,\xi_1\,Y_{41}\,X_{11} + e^{-\imath E_2 t/\hbar}\,\xi_2\,Y_{42}X_{21} + e^{-\imath E_3 t/\hbar}\,\xi_3\,Y_{4n}\,X_{31} + e^{-\imath E_4 t/\hbar}\,\xi_4\,Y_{44}\,X_{41},$$

$$Z_{12} = e^{-\imath E_1 t/\hbar}\,\xi_1\,Y_{31}\,X_{11} + e^{-\imath E_2 t/\hbar}\,\xi_2\,Y_{3n}X_{21} + e^{-\imath E_3 t/\hbar}\,\xi_3\,Y_{33}\,X_{31} + e^{-\imath E_4 t/\hbar}\,\xi_4\,Y_{34}\,X_{41},$$

$$Z_{21,n} = e^{-\imath E_1 t/\hbar}\,\xi_1\,Y_{41}\,X_{12} + e^{-\imath E_2 t/\hbar]}\,\xi_2\,Y_{42}\,X_{22} + e^{-\imath E_3 t/\hbar}\,\xi_3\,Y_{43}\,X_{32} + e^{-\imath E_4 t/\hbar}\,\xi_4\,Y_{44}\,X_{42},$$

$$Z_{22} = e^{-\imath E_1 t/\hbar}\,\xi_1\,Y_{31}\,X_{12} + e^{-\imath E_2 t/\hbar]}\,\xi_2\,Y_{32}\,X_{22} + e^{-\imath E_3 t/\hbar}\,\xi_3\,Y_{33}\,X_{32} + e^{-\imath E_4 t/\hbar}\,\xi_4\,Y_{34}\,X_{42},$$

$$Z_{31} = e^{-\imath E_1 t/\hbar}\,\xi_1\,Y_{41}\,X_{13} + e^{-\imath E_2 t/\hbar}\,\xi_2\,Y_{42}\,X_{23} + e^{-\imath E_3 t/\hbar}\,\xi_3\,Y_{43}\,X_{33} + e^{-\imath E_4 t/\hbar}\,\xi_4\,Y_{44}\,X_{43},$$

$$Z_{32} = e^{-\imath E_1 t/\hbar}\,\xi_1\,Y_{31}\,X_{13} + e^{-\imath E_2 t/\hbar}\,\xi_2\,Y_{32}\,X_{23} + e^{-\imath E_3 t/\hbar}\,\xi_3\,Y_{33}\,X_{33} + e^{-\imath E_4 t/\hbar}\,\xi_4\,Y_{34}\,X_{43},$$

$$Z_{41} = e^{-\imath E_1 t/\hbar}\,\xi_1\,Y_{41}\,X_{14} + e^{-\imath E_2 t/\hbar}\,\xi_2\,Y_{42}\,X_{24} + e^{-\imath E_3 t/\hbar}\,\xi_3\,Y_{43}\,X_{34} + e^{-\imath E_4 t/\hbar}\,\xi_4\,Y_{44}\,X_{44},$$

$$Z_{42} = e^{-\imath E_1 t/\hbar}\,\xi_1\,Y_{31}\,X_{14} + e^{-\imath E_2 t/\hbar}\,\xi_2\,Y_{32}\,X_{24} + e^{-\imath E_3 t/\hbar}\,\xi_3\,Y_{33}\,X_{34} + e^{-\imath E_4 t/\hbar}\,\xi_4\,Y_{34}\,X_{44},$$

where $Y_{ij} = \xi_j X^*_{ji}$.

We also can consider an another type of Bell-like pure initial state of two qubits

$$|\Psi(0)\rangle_A = \cos\theta|+,+\rangle + \sin\theta|-,-\rangle.$$

For this initial atomic state and vacuum cavity field the full initial state of the system is

$$|\Psi(0)\rangle = (\cos\theta|+,+\rangle + \sin\theta|-,-\rangle) \otimes |0,0\rangle. \tag{6}$$

For initial state (6) the time-dependent wave function can be written in the form

$$|\Psi(t)\langle = C_1^{(2)}(t)|+,+,0,0\rangle + C_2^{(2)}(t)|+,-,0,1\rangle + C_3^{(2)}(t)|-,+,1,0\rangle + C_4^{(2)}(t)|+,-,1,0\rangle +$$

$$+C_5^{(2)}(t)|-,+,0,1\rangle + C_6^{(2)}(t)|-,-,2,0\rangle + C_7^{(2)}(t)|-,-,0,2\rangle + C_8^{(2)}(t)|-,-,1,1\rangle + C_9^{(2)}(t)|-,-,0,0\rangle. \tag{7}$$

The coefficients $C_i(t)$ may be obtained by using the way which is described in previous case.

For two-qubit system described by the reduced density operator $\rho_A(t)$, a measure of entanglement or negativity can be defined in terms of the negative eigenvalues μ_i^- of partial transpose of the reduced atomic density matrix ($\rho_A^{T_1}$) [30, 31]

$$\varepsilon = -2\sum \mu_i^-. \tag{8}$$

When $\varepsilon = 0$ two qubits are separable and $\varepsilon > 0$ means the atom-atom entanglement. The case $\varepsilon = 1$ indicates maximum entanglement.

Using the evident form of time wave function (5) or (7) one can obtain the density operator for the whole system as

$$\rho(t) = |\Psi(t)\rangle\langle\Psi(t)|. \tag{9}$$

Taking a partial trace over the field variable one can obtain from (9) the reduced atomic density operator in the two-qubit basis $|+,+\rangle$, $|+,-\rangle$, $|-,+\rangle$, $|-,-\rangle$ for initial state (2) in the form

$$\rho_A(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & V(t) & H(t) & 0 \\ 0 & H(t)^* & W(t) & 0 \\ 0 & 0 & 0 & R(t) \end{pmatrix}. \tag{10}$$

The matrix elements of (10) are

$$V(t) = |C_4^{(1)}(t)|^2, \quad W(t) = |C_3^{(1)}(t)|^2, \quad R(t) = |C_1^{(1)}(t)|^2 + |C_2^{(1)}(t)|^2, \quad H(t) = C_4^{(1)}(t)C_3(t)^*.$$

The partial transpose of the reduced atomic density matrix (10) is

$$\rho_A^{T_1}(t) = \begin{pmatrix} 0 & 0 & 0 & H(t)^* \\ 0 & V(t) & 0 & 0 \\ 0 & 0 & W(t) & 0 \\ H(t) & 0 & 0 & R(t) \end{pmatrix}. \tag{11}$$

Matrix (11) has only one eigenvalue, which may take a negative value. As a result, in the considered case the negativity can be written from (8) as

$$\varepsilon(t) = \sqrt{R(t)^2 + 4|H(t)|^2} - R(t). \tag{12}$$

The reduced atomic density matrix ρ_A for initial state (6) has the form

$$\rho_A(t) = \begin{pmatrix} U_1(t) & 0 & 0 & H_1(t) \\ 0 & V_1(t) & H_2(t) & 0 \\ 0 & H_2(t)^* & W_1(t) & 0 \\ H_1(t)^* & 0 & 0 & R_1(t) \end{pmatrix},$$

where one can obtain with using (7)

$$U_1(t) = |C_1^{(2)}(t)|^2, \quad H_1(t) = C_1^{(2)}(t)C_9^{(2)*}(t), \quad H_2(t) = C_2^{(2)}(t)C_5^{(2)*}(t) + C_4^{(2)}(t)C_3^{(2)*}(t),$$
$$V_1(t) = |C_2^{(2)}(t)|^2 + |C_4^{(2)}(t)|^2, \quad W_1(t) = |C_3^{(2)}(t)|^2 + |C_5^{(2)}(t)|^2,$$
$$R_1(t) = |C_6^{(2)}(t)|^2 + |C_7^{(2)}(t)|^2 + |C_8^{(2)}(t)|^2 + |C_9^{(2)}(t)|^2.$$

The corresponding partial transpose of the reduced atomic density matrix $\rho_A^{T_1}$ is

$$\rho_A(t)^{T_1} = \begin{pmatrix} U_1(t) & 0 & 0 & H_2(t)^* \\ 0 & V_1(t) & H_1(t)^* & 0 \\ 0 & H_1(t) & W_1(t) & 0 \\ H_2(t) & 0 & 0 & R_1(t) \end{pmatrix}. \tag{13}$$

Matrix (13) has two eigenvalues, which may take a negative value. Then, the negativity can be written as a superposition of two terms. At the same time, each term contributes to the total amount, as long as it takes a positive value. As a result the negativity from (8) is

$$\varepsilon(t) = \sqrt{(U_1(t) - R_1(t))^2 + 4|H_2(t)|^2} - U_1(t) - R_1(t) + \sqrt{(V_1(t) - W_1(t))^2 + 4|H_1(t)|^2} - V_1(t) - W_1(t). \tag{14}$$

3. Model includes Jaynes-Cummings qubit and trapped qubit

Let us consider the other types of JCM with dipole-dipole interaction and detuning. The first of them is the following. We have two identical superconducting qubits and one-mode quantum electromagnetic cavity field. The first qubit A is trapped in a lossless microcavity and nonresonantly interacts with the cavity field. The second qubit B lies beside the first qubit out of the cavity. We assume that the direct dipole-dipole interaction between qubits takes place. In a frame rotating with the field frequency, the Hamiltonian for the system under rotating wave approximation can be written as

$$H = (1/2)\hbar\delta\sigma_A^z + \hbar\gamma(\sigma_A^+ a + a^+\sigma_A^-) + \hbar J(\sigma_A^+\sigma_B^- + \sigma_A^-\sigma_B^+), \tag{15}$$

We use notations as in Section 2. The time-dependent wave-function for model with Hamiltonian (15) and initial state (2) has the following form

$$|\Psi(t)\rangle = C_1^{(3)}(t)|+,-,0\rangle + C_2^{(3)}(t)|+,-,0\rangle + C_3^{(3)}(t)|+,-,1\rangle. \tag{16}$$

The evident form of coefficients $C_i^{(3)}(t)$ may be easily obtained by using the way which is described in previous case. For model under consideration the atomic density operator in the two-qubit basis $|+,+\rangle$, $|+,-\rangle$, $|-,+\rangle$, $|-,-\rangle$ for initial state (2) can be written in the form (10), where

$$V(t) = |C_1^{(3)}(t)|^2, \quad W(t) = |C_2^{(3)}(t)|^2, \quad R(t) = |C_3^{(3)}(t)|^2, \quad H(t) = C_1^{(3)}(t)C_2^{(3)}(t)^*.$$

Therefore the negativity can be written in the form (8).

4. Two-qubits Jaynes-Cummings model

At last, we consider two-atom JCM. We have two identical superconducting qubits A and B non-resonantly interacting with common one-mode quantum electromagnetic field of coplanar resonator. As in previous cases we assume that the direct dipole-dipole interaction between qubits takes place. In a frame rotating with the field frequency, the Hamiltonian for the system under rotating wave approximation can be written as

$$H = \hbar\delta\sigma_A^z + \hbar\delta\sigma_B^z + \hbar\gamma\sum_{i=A}^{B}(\sigma_i^+ a + a^+\sigma_i^-) + \hbar J(\sigma_A^+\sigma_B^- + \sigma_A^-\sigma_B^+), \tag{17}$$

We use notations as in Section 2 and 3. The time-dependent wave-function for model with Hamiltonian (16) and initial state (2) has the form (16). The exact solution for coefficients $C_i^{(3)}(t)$ may be easily obtained by using the dressed-states representations as in previous Section 2 and 3. Therefore the negativity for considered model can be written in the form (8).

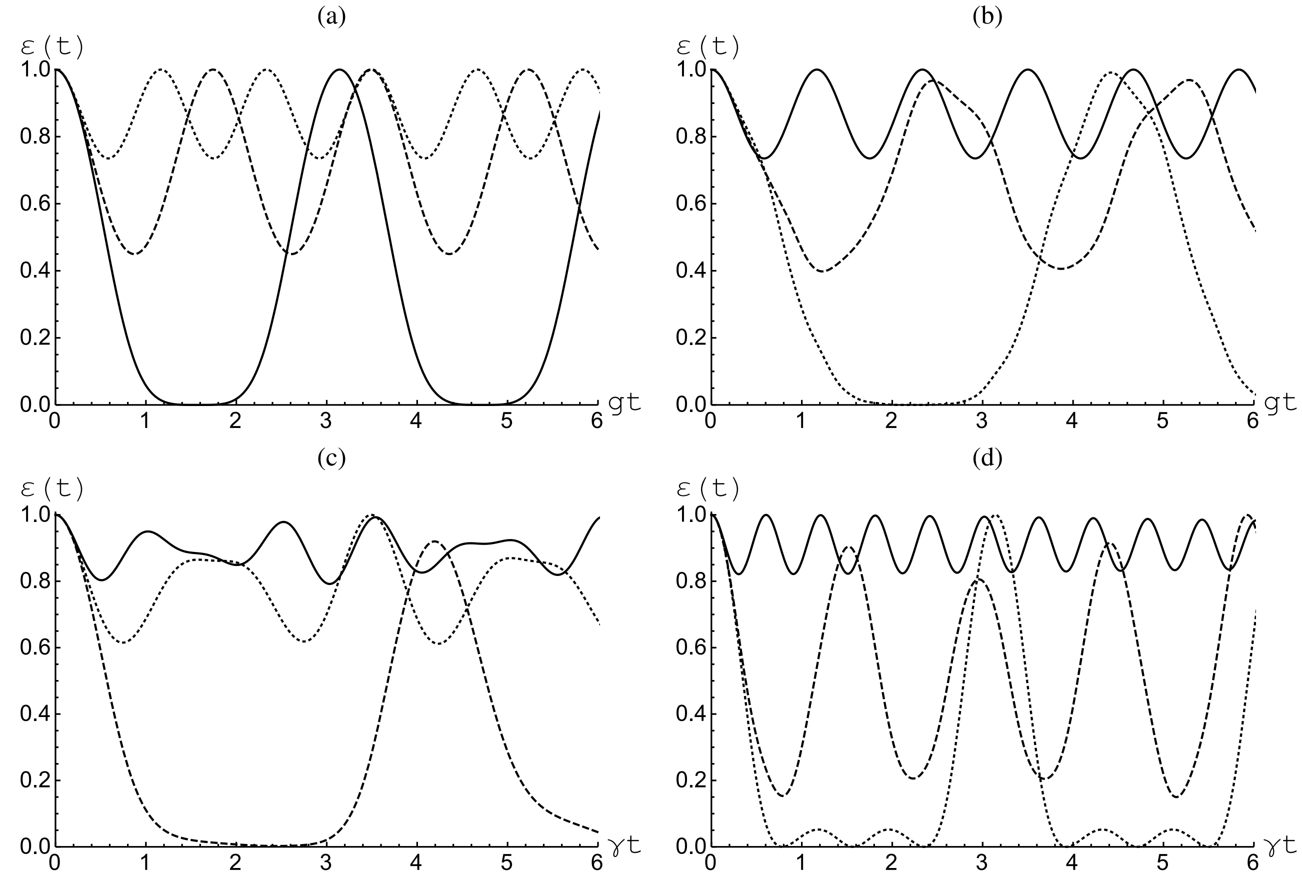

Fig. 1. The negativity as a function of a scaled time gt ($g \equiv g_a$) for double JCM and initial state (2) and $\delta_a = \delta_b = 0$, $\gamma_b = \gamma_a$ (a), $\delta_a = -\delta_b = 5$, $\gamma_b = \gamma_a$ (b), $\delta_a = \delta_b = 5$, $\gamma_b = \gamma_a$ (c) and $\delta_a = \delta_b = 0$, $\gamma_a = 2\gamma_b$ (d). Parameter $\theta = \pi/4$. The strength of dipole-dipole interaction $\alpha = 0$ (dotted), $\alpha = 3$ (dashed) and $\alpha = 5$ (solid).

5. Modeling of qubits entanglement dynamics

The results of calculations of entanglement parameter (12) for double JCM and initial atomic states (2) are shown in Fig. 1(a)-(d) and these for initial atomic state (6) are displayed in Fig. 2(a-d). Fig. 1(a) shows that in the case of exact resonance, the dependence of the negativity evolves periodically between 0 and 1, but the period is affected by the different coupling constants between the qubits and cavities. In this case the inclusion of the dipole-dipole interaction leads to a stabilization of entanglement behavior. Figs. 1(b)-1(d) show the effect of dipole-dipole interaction on negativity for non-resonant interaction and different couplings. When qubits A and B interact with a single-mode cavity fields via not-zero detuning (or coupling are nonequal) the presence of dipole-dipole interaction with intermediate strength leads to increasing of the amplitudes of the negativity oscillations. But for large values of dipole-dipole interaction strength one can see the stabilization of entanglement oscillations as in the case of exact resonance. Figs. 2(a) and Fig. 2(d) show the time dependence of negativity (14) for initial qubits state (6) and different strength of dipole-dipole interaction in the case of exact resonance. This is different from the results obtained for the previous case. The dipole-dipole interaction in the present case does not lead to stabilization of the entanglement, but has only an effect on the periods and amplitudes of the oscillations of entanglement. However, for non resonant interaction between qubits and fields the influence of dipole-dipole interaction on the entanglement is opposite to the previous case. For large values of the dipole-dipole interaction strength we have to deal with the stabilization of entanglement. Figs. 3 and 4 show the influence of detuning on the negativity behavior for one-atom and two-atom JCM and initial entangled state (2). For these models the influence of dipole-dipole interaction on entanglement is similar to double JCM model.

6. Conclusion

In this paper, we investigate the entanglement between two superconducting qubits interacting with microwave fields of coplanar resonators in the framework three type of JCM: double JCM with different coupling constants and detunings, one-qubit and two-qubit with detunings taking into account the direct dipole-dipole interaction, and discuss dependence of the dipole-dipole interaction on qubit-qubit entanglement for resonance and nonresonance interactions. The results show that these parameters have great impact on the amplitude and the period of the atom-atom entanglement evolution. In addition, the presence of sufficiently large dipole-dipole interaction leads to stabilization of entanglement for all Bell-types initial qubits states and different couplings and detunings. In this paper we have investigated the dynamics of entanglement for lossless cavity. We will consider the entanglement behavior for finite-Q cavity in following paper by using the master equation derived in [32, 33].

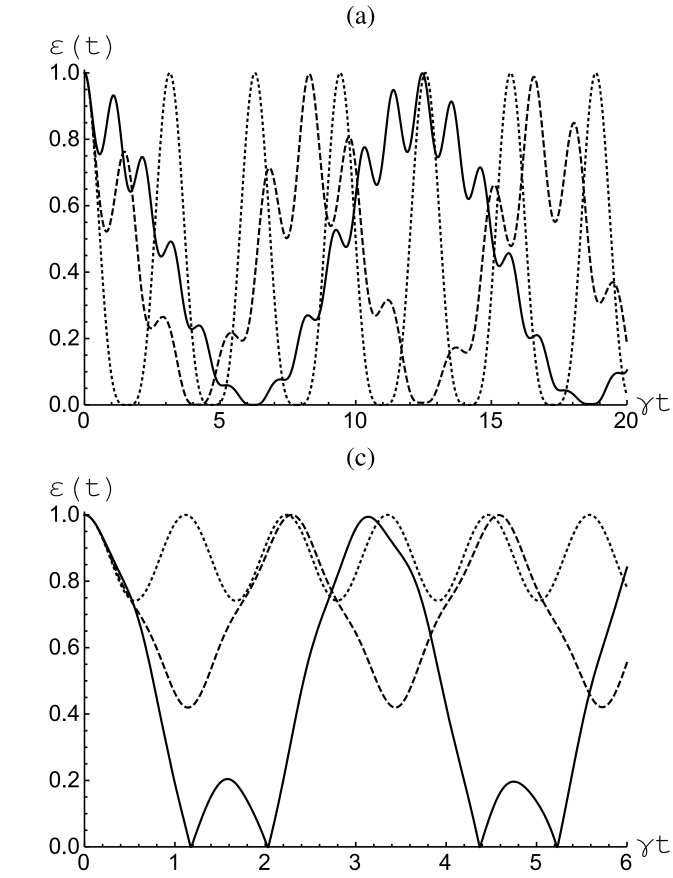

Fig. 2. The negativity as a function of a scaled time gt ($g \equiv g_a$) for double JCM and initial state (6) and $\delta_a = \delta_b = 0$, $\gamma_b = \gamma_a$ (a), $\delta_a = -\delta_b = 5$, $\gamma_b = \gamma_a$ (b), $\delta_a = \delta_b = 5$, $\gamma_b = \gamma_a$ (c) and $\delta_a = \delta_b = 0$, $\gamma_a = 2\gamma_b$ (d). Parameter $\theta = \pi/4$. The strength of dipole-dipole interaction $\alpha = 0$ (dotted), $\alpha = 3$ (dashed) and $\alpha = 5$ (solid).

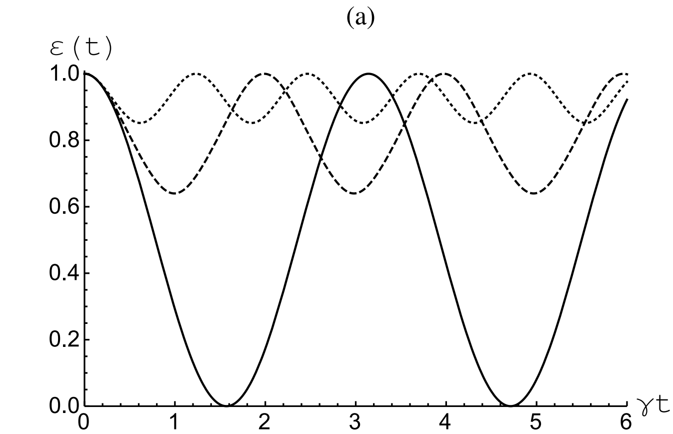

Fig. 3. The negativity as a function of a scaled time gt ($g \equiv g_a$) for one-atom JCM and initial state (2) and $\delta = 0$ (a), $\delta = 5$ (b). Parameter $\theta = \pi/4$. The strength of dipole-dipole interaction $\alpha = 0$ (dotted), $\alpha = 3$ (dashed) and $\alpha = 5$ (solid).

References

[1] Nielsen, M.A. Quantum Computation and Quantum Information / M.A Nielsen, I.L. Chuang – Cambrige: Cambrige University Press, 2000. – 700 p.

[2] Orlando, T.P. Superconducting persistent-current qubit / T.P. Orlando et al. // Phys. Rev. – 1999. – Vol. B60. – P. 15398

[3] Scully, M.O. Quantum optics / M.O. Scully, M.S. Zubairy – Cambridge: Cambridge University Press, 1997. – 630 p.

[4] Buluta, I. Natural and artificial atoms for quantum computation / I. Buluta, S. Ashhab, F. Nori // Rep. Prog. Phys. – 2001. – Vol. 74. – P. 104401.

[5] Bashkirov, E.K. Dynamics of the Two-Atom Jaynes-Cummings Model with Nondegenerate Two-Photon Transitions / E.K. Bashkirov // Laser Physics – 2006. – Vol. 16. – P. 1218-1226.

[6] Bashkirov, E.K. Squeezing and amplitude-squared squeezing in the model of two nonidentical two-level atoms / E.K. Bashkirov // International Journal of Modern Physics – 2007.– Vol. B21.– P. 145-157 (2007).

[7] Bashkirov, E.K., "Entanglement in the degenerate two-photon Tavis-Cummings model / E.K. Bashkirov // Physica Scripta – 2010. – Vol. 82. – P. 015401.

[8] Yönac, M.Y. Sudden death of entanglement of two JaynesCummings atoms / M.Y. Yönac, T. Yu, J.H. Eberly // J. Phys. B: At. Mol. Opt. Phys. – 2006. – Vol. – 39. – P. S621-S625.

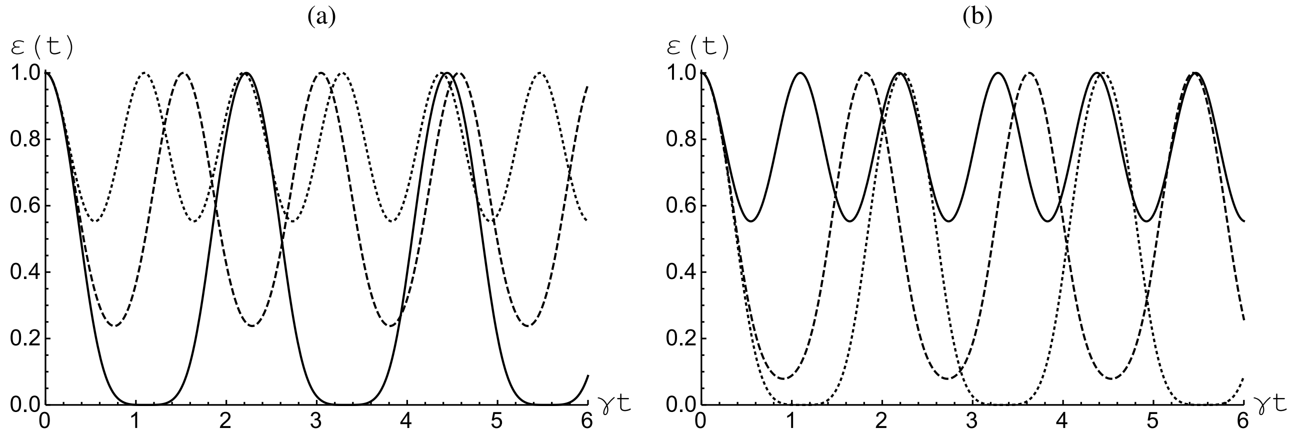

Fig. 4. The negativity as a function of a scaled time gt ($g \equiv g_a$) for two-atom JCM and initial state (2) and $\delta = 0$ (a), $\delta = 5$ (b). Parameter $\theta = \pi/4$. The strength of dipole-dipole interaction $\alpha = 0$ (dotted), $\alpha = 3$ (dashed) and $\alpha = 5$ (solid).

[9] Yönac, M.Y. Pairwise concurrence dynamics: a four-qubit model / M.Y. Yönac, T. Yu, J.H. Eberly // J. Phys. B: At. Mol. Opt. Phys. – 20077. – Vol. 40. P. S45-S59.

[10] Hu, Y.-H. Effect of the Stark shift on entanglement in a double two-photon JC model / Y.-H Hu M.-F. Fang, J.-W. Cai, K. Zeng, C.-L. Jiang // J. Mod. Opt. – 2008. Vol. 55(21). P. 3551-3562.

[11] Hu, Y.-H. Sudden Death and Long-Lived Entanglement Between Two Atoms in a Double JC Model System / Y.-H Hu M.-F. Fang, J.-W. Cai, K. Zeng, C.-L. Jiang // Int. J. Theor. Phys. – 2008. – Vol. 47. P. 2554-2565.

[12] Du, M. Sudden birth of entanglement between two atoms in a double JC model / M. Du, M.-F. Fang, X. Liu // Chin. Opt. Lett. – 2009. Vol 7(5). – P. 443-445.

[13] Xie, Q. Entanglement Dynamics of the Double Intensity-Dependent Coupling Jaynes-Cummings Models / Q. Xie, M.-F. Fang, // Int. J. Theor. Phys. – 2012. – Vol. 51. – P. 778-786.

[14] Liao, Q. The Entanglement Dynamics of Two Atoms in a Double Two-PhotonJaynes-Cummings Model / Q. Liao, W. Nie, N. Zhou, Y. Liu, M.A. Ahmad // Chin. J. Phys. – 2013. – Vol. 51(2). – P. 404-411.

[15] Xie, Q. Entanglement Dynamics of Atoms in Double Jaynes-Cummings Models with Kerr Medium / Q. Xie, M.-F. Fang // Commun. Theor. Phys. – 2010. Vol. 54(5). – P. 840-844.

[16] Ouyang, X.-C.Entanglement dynamics of a double two-photon Jaynes Cummings model with Kerr-like medium / X.-C. Ouyang, M.-F. Fang, G.-D. Kang, X.-J Deng, L.-Y. Huang, // Chin. Phys. – 2010. Vol. B19(3). – P. 030309(1-6).

[17] Vieira, A.R. Geometry in the Entanglement Dynamics of the Double Jaynes-Cummings Model / A.R. Vieira, J.G.G. de Oliveira Junior, J.G. Peixoto de Faria, M.C. Nemes // Braz. J. Phys. – 2014. – Vol. 44. – P. 19-29.

[18] Baghshahi, H.R. Entanglement Criteria of Two Two-Level Atoms Interacting with Two Coupled Modes / H.R. Baghshahi, M.Z. Tavassoly, M.J. Faghihi // Int. J. Theor. Phys. – 2015. – Vol. 54(8). – P. 2839-2854.

[19] Zou, H.-M. Analytical solution and entanglement swapping of a double JaynesCummings model in non-Markovian environments / H.-M. Zou, M.-F. Fang // Quan. Inf. Process. – 2015. – Vol. 14. – P. 2673-2686.

[20] Zhu, W.-T. Entanglement Dynamics of Two Qubits Coupled Independently to Cavities in the Ultrastrong Coupling Regime: Analytical Results / W.-T. Zhu, Q.-B. Ren, L.-W. Duan, Q.-H. Chen // Chin. Phys. Lett. – 2016. – Vol. 33(5). – P. 050302(1-4).

[21] Izmalkov, A. Evidence for Entangled States of Two Coupled Flux Qubits / A. Izmalkov et.al. // Phys. Rev. Lett. – 2004. – Vol. 93. – P. 037003.

[22] Grajcar, M. Direct Josephson coupling between superconducting flux qubits / M. Grajcar et al. // Phys. Rev.– 2005. Vol. B72. – 020503(R).

[23] Majer, J.B. Spectroscopy on two coupled flux qubits / J.B. Majer et al. // Phys. Rev. Lett. – 2005. – Vol. 94. – 090501.

[24] Bashkirov, E.K., Mastyugin, M.S., "Entanglement of two superconducting qubits interacting with two-mode thermal field / E.K. Bashkirov, M.S. Mastyugin // Computer Optics – 2013. – Vol. 37(3). – P. 278-285.

[25] Bashkirov, E.K.The influence of the dipole-dipole interaction and atomic coherence on the entanglement of two atoms with degenerate two-photon transitions / E.K. Bashkirov, M.S. Mastyugin // Optics and Spectroscopy – 2014. – Vol. 116(4). – P. 630-634.

[26] Bashkirov, E.K. The dynamics of entanglement in two-atom Tavis-Cummings model with non-degenerate two-photon transitions for four-qubits initial atom-field entangled states/ E.K. Bashkirov, M.S. Mastyugin // Optics Communications – 2014. – Vol. 313. – P. 170-174.

[27] Bashkirov, E.K. The influence of atomic coherence and dipole-dipole interaction on entanglement of two qubits with nondegenerate two-photon transitions / E.K. Bashkirov, M.S. Mastyugin // Pramana - Journal of Physics – 2015. – Vol. 84(1). – P. 127-135.

[28] Bashkirov, E.K. Entanglement Between Qubits Interacting with Thermal Field / E.K. Bashkirov, M.S. Mastyugin // EPJ Web of Conferences – 2015. – Vol. 103. P. 03002.

[29] Bashkirov, E.K. Entanglement between two qubits induced by thermal field / E.K. Bashkirov, M.S. Mastyugin // Journal of Physics: Conference Series – 2016. – Vol. 735. – P. 012025.

[30] Peres, A. Separability Criterion for Density Matrices / A. Peres // Phys. Rev. Lett. – 1996. – Vol. 77. – P. 1413-1415.

[31] Horodecki, R. Separability of Mixed States: Necessary and Sufficient Condition / R.Horodecki, M. Horodecki, P. Horodecki // Phys.Lett. – 1996. – Vol. A223. P. 333-339.

[32] Bashkirov, E.K. Dynamics of two two-level atoms in finite-Q cavity / E.K. Bashkirov, E.G. Mangulova // Izvestiya Akademii Nauk. Seriya Fizicheskaya – 2000. – Vol. 64(10). – P. 2075-2079.

[33] Bashkirov, E.K. Spontaneous emission of two three-level atoms in nonideal cavity / E.K. Bashkirov // Izvestiya Akademii Nauk. Seriya Fizicheskaya – 2004. – Vol. 68(9). – P. 1292-1295.