

SAMARA UNIVERSITY

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Thermodynamic calculation of the turbofan engine

Methodical instructions for course work

CAMAPA 2017

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Task

Evaluate the ideal GTE cycle using initial data given in Table 1.

Table 1 – Initial data

Air composition				
H, m	N ₂ , %	O ₂ , %	CO ₂ , %	H ₂ O, %
10000	78,01	20,87	0,11	1,01
Fuel composition and properties				
	Chemical formula	Sulfur and moisture percentage, %	Density at 20°C, kg/m ³	Low heat value H _u , kJ/kg
T – 1	C _{7,2} H _{13,3}	0.005	0.775	43000
Air physical properties (as a function of altitude)				
H _n , m	T ₀ , K	P ₀ , N/m ²	ρ kg/m ³	
10000	218	17000		
Additional values				
Engine thrust, R, N		160000		
Turbine inlet temperature T ₃ , K		1630		
Fuel consumption rate q, kg/(h·kN)		0.0595		
Ambient air temperature T _T , K		300		
Flight velocity mach number, km/h		0.8		

List of symbolic notations and indices

- C_0 — incident air flow rate, m/s
 C_5 — exhaust gas rate, m/s
 C_p — heat capacity at constant pressure, J/kg·K
 C_v — heat capacity at constant volume, J/kg·K
 G — mass, kg
 H — altitude, m
 k — ratio of specific heats
 M — molar mass, mol
 p — pressure, Pa
 q — heat, J/kg
 R — specific gas constant, J/(mol·K)
 R — universal gas constant, J/(kg·K)
 $R_{y\theta}$ — specific thrust, m/s
 L — specific work;
 S — entropy, J/kg
 T — temperature, K
 U — internal energy, J/kg
 v — specific volume, m³/kg
 α — air-to-fuel ratio
 Δ — change of parameter
 η_t — cycle thermal efficiency, %
 ρ_0 — air density, kg/m³
 τ — time, h
' — superscript of air parameters
" — superscript of combustion gases parameters
 opt — optimum;
 i — component/process subscript;
 ц — cycle;
 к — compressor;
O — point O in process diagram.

Introduction

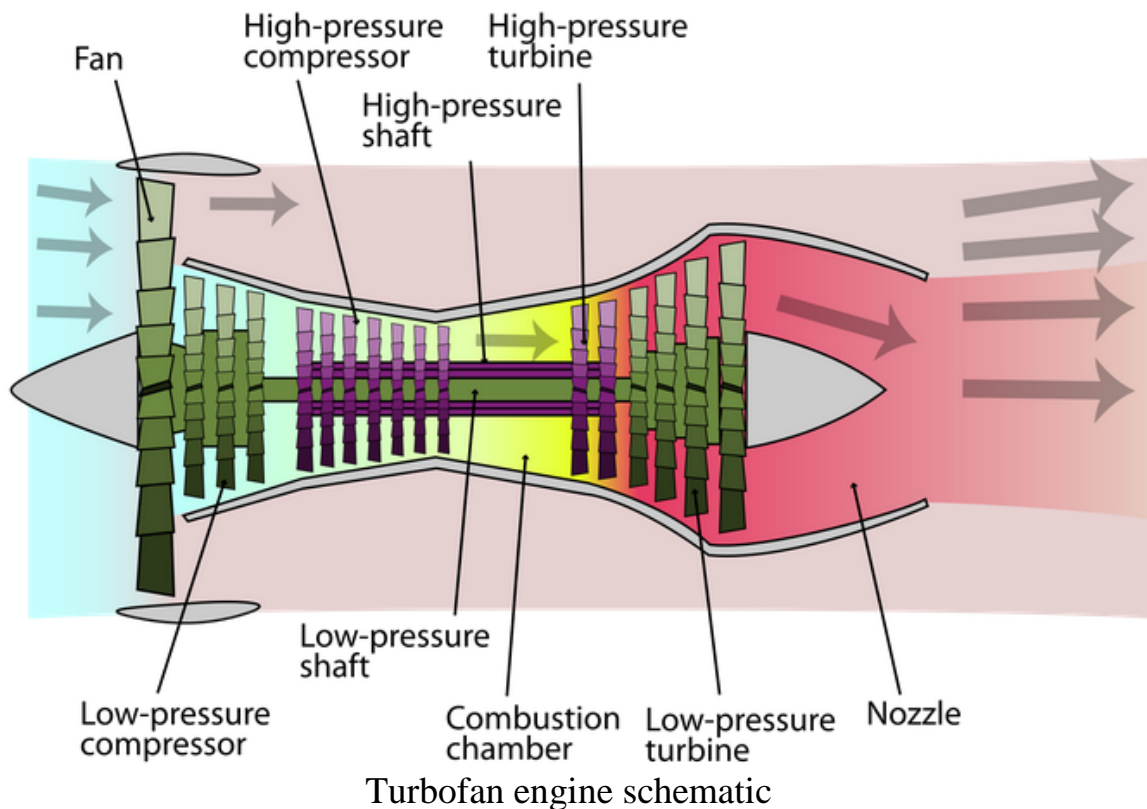
Aircraft gas turbine engine (GTE) is a complex technological system with high efficiency. Its construction has been being improved to perfection through many experiments and by using statistical data. All technological achievements in spheres of construction, material science, manufacturing, and methods of increasing engine load capacitance and fatigue limit has been realized in modern engine. New generation engines have been developed and successfully manufactured with some implemented modifications which have enhanced specific operation parameters. Large volume of useful data has been collected from practical use and development of GTE which are still being used, maintained and manufactured. This data is of great help to engineers who work on improving GTE characteristics and designing new engine constructions of present and future generations.

Factors which define the quality of aircraft engine are the following: quality of engine construction and materials; manufacturing processes; technological processes of assembly and test of components and engine itself; quality of spare parts; measurement assurance of production processes; storage/retrieval operations; operating conditions and others.

The objective of the course work is evaluation of working medium state parameters and energy characteristics of turbofan engine. Calculations should be made for the ideal turbofan engine cycle, considering heating process to be isobaric.

1. Brief description of turbofan engine operating principle

Turbofan engine is a high-by-pass-ratio engine (by-pass ratio is higher than 2). A single stage fan of large diameter is used in such engines maintaining high air engine flow rate at all flight velocities, including low takeoff and landing velocities. As the fan diameter of by-pass GTE is quite large, the jet nozzle is made shortened in order to decrease its weight. Nozzle is equipped with straightener blades which turn airflow axially. That is the reason why the majority of by-pass GTE are without flow mixing. Economic efficiency of turbofan engine is due to additional thrust made by the fan which rotational energy is converted from jet exhaust stream energy. Turbofan engine can produce 70-80% of ideal turbofan engine thrust.



2. Evaluation of working medium cycle

2.1 Evaluation of working medium composition

Evaluation of mass and mole components fractions, molecular mass and heat capacity is made for air which enters GTE at the altitude and velocity given.

Volume component fractions are:

$$r_{N_2} = 0,7753$$

$$r_{O_2} = 0,2039$$

$$r_{CO_2} = 0,0059$$

$$r_{H_2O} = 0,0149$$

Molecular mass of mixture is found as:

$$\mu_{CM} = \sum_{i=1}^4 r_i \cdot \mu_i$$
$$\mu_{CM} = r_1 \cdot \mu_1 + r_2 \cdot \mu_2 + r_3 \cdot \mu_3 + r_4 \cdot \mu_4$$

Mass fractions of each component at the air of 1 kg mass is found as:

$$g_1 = \frac{r_1 \cdot \mu_1}{\mu_{CM}}; g_2 = \frac{r_2 \cdot \mu_2}{\mu_{CM}}; g_3 = \frac{r_3 \cdot \mu_3}{\mu_{CM}}; g_4 = \frac{r_4 \cdot \mu_4}{\mu_{CM}}$$

The results should obey the following equation:

$$\sum_{i=1}^4 g_i = 1;$$

Mole fraction and mass of each component at the mixture of 1 kg is found as:

$$G_i = g_i \cdot G_{CM},$$

$$G_{CM} = 1 \text{ kg},$$

$$M_i = \frac{G_i}{\mu_i}$$

$$M_1 = \frac{G_1}{\mu_1}; M_2 = \frac{G_2}{\mu_2}; M_3 = \frac{G_3}{\mu_3}; M_4 = \frac{G_4}{\mu_4}$$

The next step is calculating isobaric and isochoric heat capacities, and ratio of specific heats and specific gas mixture constant:

$$C'_p = \sum_{i=1}^4 g'_i \cdot C'_{pi};$$

$$C'_v = \sum_{i=1}^4 g'_i \cdot C'_{vi};$$

$$C'_p = g'_1 \cdot C'_{p1} + g'_2 \cdot C'_{p2} + g'_3 \cdot C'_{p3} + g'_4 \cdot C'_{p4}$$

$$C'_v = g'_1 \cdot C'_{v1} + g'_2 \cdot C'_{v2} + g'_3 \cdot C'_{v3} + g'_4 \cdot C'_{v4}$$

$$k' = \frac{C'_p}{C'_v}$$

$$R' = C'_p - C'_v$$

The result should obey the following equation:

$$R_{CM} = \frac{8314}{\mu_{CM}}$$

2.2 Evaluation of optimum GTE compression ratio

The first step is to define air temperature after the diffusor. When air flow passes the fan its pressure increases. Fan pressure ratio lies between 1.5 and 2. Air temperature after the fan is defined as:

$$T_1 = T_0 \cdot \pi_g^{\frac{k-1}{k}}$$

π_B – compressor pressure ratio.

Optimum compressor pressure ratio equals:

$$(\pi_k)_{opt} = \left[\frac{T_3}{T_1} \right]^{\frac{k'}{2(k'-1)}}$$

2.3 Calculating air-to-fuel ratio

Fuel is kerosene TC – 1

$$j = \frac{n}{m};$$

$$f = 4 \cdot j + 1;$$

$$F = G'(O_2) \cdot \left(\frac{11 \cdot j \cdot C'_p(CO_2)}{2 \cdot f} + \frac{9 \cdot C'_p(H_2O)}{8 \cdot f} - C'_p(O_2) \right)$$

$$T_T = 300K;$$

$$H_u = 43130 \cdot 10^3 \frac{J}{kg \cdot K};$$

$$\alpha = \frac{G'(O_2) \cdot \frac{12 \cdot j + 1}{8 \cdot f} \cdot H_u - F \cdot (T_3 - T_T)}{C'_p \cdot G' \cdot \left(T_3 - T_1 \cdot \pi_k^{\frac{k'-1}{k'}} \right)};$$

2.4 Evaluating compositions of combustion gases and air-to-fuel mixture

Masses and mole and mass fractions of combustion gases components is found as:

$$\begin{aligned}
 G''(H_2O) &= G'(H_2O) + \frac{9 \cdot G'(O_2)}{8 \cdot \alpha \cdot f} \\
 G''(N_2) &= G'(N_2) \\
 G''(CO_2) &= G'(CO_2) + \frac{11 \cdot j \cdot G'(O_2)}{2 \cdot \alpha \cdot f} \\
 G''(O_2) &= \frac{\alpha - 1}{\alpha} \cdot G'(O_2) \\
 G'' &= G''(H_2O) + G''(N_2) + G''(CO_2) + G''(O_2) \\
 g''(N_2) &= \frac{G''(N_2)}{G''} \\
 g''(O_2) &= \frac{G''(O_2)}{G''} \\
 g''(CO_2) &= \frac{G''(CO_2)}{G''} \\
 g''(H_2O) &= \frac{G''(H_2O)}{G''} \\
 \sum_{i=1}^4 g_i'' &\approx 1.
 \end{aligned}$$

Fuel mass is defined as:

$$\begin{aligned}
 G_T &= \frac{G'(O_2) \cdot (12 \cdot j + 1)}{8 \cdot \alpha \cdot f} \\
 G_T + G' &= G'' ,
 \end{aligned}$$

The next step is calculating isobaric and isochoric heat capacities, and ratio of specific heats and specific combustion gases constant:

$$\begin{aligned}
 C_{pi}'' &= \sum_{i=1}^4 g_i'' \cdot C_{pi}''; \\
 C_{vi}'' &= \sum_{i=1}^4 g_i'' \cdot C_{vi}''; \\
 C_p'' &= g''(N_2) \cdot C_p'(N_2) + g''(O_2) \cdot C_p'(O_2) + g''(CO_2) \cdot C_p'(CO_2) + g''(H_2O) \cdot C_p'(H_2O); \\
 C_v'' &= g''(N_2) \cdot C_v'(N_2) + g''(O_2) \cdot C_v'(O_2) + g''(CO_2) \cdot C_v'(CO_2) + g''(H_2O) \cdot C_v'(H_2O); \\
 k'' &= \frac{C_p''}{C_v''} \\
 R'' &= C_p'' - C_v''
 \end{aligned}$$

Specific constants should be approximately equal: $R'' \approx R'$

Table 2. Working medium composition

Parameters		Components			
		N ₂	O ₂	CO ₂	H ₂ O
$R_i, \frac{J}{kg \cdot K}$		297.2	260	188.8	461
$C_{pi}, \frac{J}{kg \cdot K}$		1039.2	915	814.8	1859
$C_{vi}, \frac{J}{kg \cdot K}$		742	655	626	1398
$\mu, \frac{kg}{kmol}$		28	32	44	18
G _i , kg	Воздух				
	Прод.сгор				
M _i , mol	Воздух				
	Прод.сгор				
g _i	Воздух				
	Прод.сгор				

Table 3. Working medium properties
2.5 Cycle parameters calculation

Смесь	$C_p, \frac{J}{kg \cdot K}$	$C_v, \frac{J}{kg \cdot K}$	$R, \frac{J}{kg \cdot K}$	k	G, kg
Воздух					
Прод. сгор					

Point 1. Process 0-1 – adiabatic compression of air in the fan

$$T_1 = 300K \quad (\text{found above});$$

$$P_1 = P_0 \cdot \pi_e$$

$$\nu_0 = \frac{1}{\rho};$$

$$\nu_1 = \nu_0 \cdot \left(\frac{P_0}{P_1} \right)^{\frac{1}{k}}.$$

Point 2. Process 1-2 – adiabatic compression of air in the compressor

$$T_2 = T_1 \cdot (\pi_k)_{Opt}^{\frac{k'-1}{k'}}$$

$$P_2 = P_1 \cdot (\pi_k)_{Opt}$$

$$v_2 = \frac{R \cdot T_2}{P_2}$$

Point 3. Process 2-3 – isobaric heating in the combustion chamber

$$T_3;$$

$$P_3 = P_2;$$

$$\rho = \frac{T_3}{T_2} - \text{temperature ratio};$$

$$v_3 = \frac{R \cdot T_3}{P_3}$$

Point 4. Process 3-4 – adiabatic expansion of working mixture in the turbine

$$T_4 = T_3 - T_2 + T_1$$

$$P_4 = P_3 \cdot \left(\frac{T_4}{T_3} \right)^{\frac{k'}{k'-1}}$$

$$v_4 = \frac{R \cdot T_4}{P_4}$$

Point 6. Process 4-6 – adiabatic expansion of working mixture in the nozzle down to ambient pressure $p_0 = p_5$

$$P_6 = P_0$$

$$T_6 = T_4 \cdot \left(\frac{P_0}{P_3} \right)^{\frac{k'-1}{k'}}$$

$$v_6 = \frac{R \cdot T_6}{P_6}$$

2.6 Calculation of fan drive turbine power

The next step is defining turbine power which is enough large to drive the fan. It can be derived from thrust equation of by-pass engine:

$$R = G_g (c_6 - c_0) + F_{aem} (p_1 - p_0).$$

Thrust-airflow-rate ration is defined as:

$$\frac{R}{G_g} = (c_6 - c_0) + \frac{F_{aem} (p_1 - p_0)}{G_g}$$

Inlet flow rate can be defined using specific fuel consumption rate:

$$G_m = q_m \cdot R$$

$$G(O_2) = \frac{8 \cdot \alpha \cdot f \cdot G_T}{(12 \cdot j + 1)}$$

$$G_g = \frac{G(O_2)}{g_2}$$

Mass inlet rate equals:

$$G_g = \rho_0 c_0 F_0$$

Air velocity is found as:

$$c_0 = M \cdot a = M \cdot \sqrt{k \cdot R \cdot T_0}$$

$$\frac{R}{G_g} = (c_6 - c_0) + \frac{F_{\text{вент}} p_0 (\pi_g - 1)}{\rho_0 c_0 F_0}$$

By-pass ratio is a ratio of fan flow area to inlet flow area:

$$m = \frac{G_{\text{вент}}}{G_g} = \frac{\rho_0 F_{\text{вент}} c_0}{\rho_0 F_g c_0} = \frac{F_{\text{вент}}}{F_g}$$

Derived from ideal gas flow is:

$$\frac{p}{\rho} = RT$$

Then

$$\frac{R}{G_g} = (c_6 - c_0) + \frac{m R T_0 (\pi_g - 1)}{c_0} \cdot *$$

Fan power is defined as:

$$N_{\text{вент}} = G_{\text{вент}} \cdot \frac{k}{k-1} R T_0 \left[\pi_g^{\frac{k-1}{k}} - 1 \right]$$

Divided by G_B :

$$l_{\text{вент}} = m \frac{k}{k-1} R T_0 \left[\pi_g^{\frac{k-1}{k}} - 1 \right]$$

Then, by-pass ratio is defined as:

$$m = \frac{l_{\text{вент}}}{\frac{k}{k-1} R T_0 \left[\pi_g^{\frac{k-1}{k}} - 1 \right]} \cdot **$$

Value of velocity c_6 is derived from:

$$i_5 = i_6 + \frac{c_6^2}{2}$$

$$i_5 = i_4 - l_{\text{вент}}$$

$$c_6 = \sqrt{2(i_4 - i_6 - l_{\text{вент}})} = \sqrt{2(c_p (T_4 - T_6) - l_{\text{вент}})} \cdot ***$$

Substituting equations ** and *** to *:

$$\frac{R}{G_g} = \sqrt{2(c_p (T_4 - T_6) - l_{\text{вент}})} - c_0 + \frac{l_{\text{вент}} (\pi_g - 1)}{c_0 \frac{k}{k-1} \left(\pi_g^{\frac{k-1}{k}} - 1 \right)}$$

Solution of the equation above is quite sophisticated to be derived analytically, despite it has only one indeterminate ($l_{\text{вент}}$). It is recommended to use Mathcad program to solve it analytically or diagrammatically.

Having found fan work value, by-pass ratio can be found as:

$$m = \frac{l_{\text{genm}}}{\frac{k}{k-1} RT_0 \left[\pi_6^{\frac{k-1}{k}} - 1 \right]}$$

The next step is defining temperature and pressure:

$$T_5 = \frac{i_5}{c_p} = \frac{i_4 - l_{\text{genm}}}{c_p} = T_4 - \frac{l_{\text{genm}}}{c_p}.$$

$$P_5 = P_4 \cdot \left(\frac{T_5}{T_4} \right)^{\frac{k}{k-1}}$$

3. Calculating caloric values of GTE cycle

3.1 Changes of caloric values at cycle processes

Internal energy change, enthalpy change and entropy change can be found from:

$$\Delta U_i = C_v \cdot (T_{i+1} - T_i); \quad \Delta h_i = C_p \cdot (T_{i+1} - T_i); \quad \Delta S_i = C_p \cdot \ln \left(\frac{T_{i+1}}{T_i} \right) - R \cdot \ln \left(\frac{P_{i+1}}{P_i} \right);$$

$$\Delta U_{0-1} = C_v \cdot (T_1 - T_0)$$

$$\Delta U_{1-2} = C_v \cdot (T_2 - T_1)$$

$$\Delta U_{2-3} = C_v \cdot (T_3 - T_2)$$

$$\Delta U_{3-4} = C_v \cdot (T_4 - T_3)$$

$$\Delta U_{4-5} = C_v \cdot (T_5 - T_4)$$

$$\Delta U_{5-6} = C_v \cdot (T_6 - T_5)$$

$$\Delta U_{6-0} = C_v \cdot (T_0 - T_6)$$

$$\Delta U_{\Sigma} = \sum_{i=1}^7 \Delta U_i$$

$$\Delta h_{0-1} = C_p \cdot (T_1 - T_0)$$

$$\Delta h_{1-2} = C_p \cdot (T_2 - T_1)$$

$$\Delta h_{2-3} = C_p \cdot (T_3 - T_2)$$

$$\Delta h_{3-4} = C_p \cdot (T_4 - T_3)$$

$$\Delta h_{4-5} = C_p \cdot (T_5 - T_4)$$

$$\Delta h_{5-6} = C_p \cdot (T_6 - T_5)$$

$$\Delta h_{6-0} = C_p \cdot (T_0 - T_6)$$

$$\Delta h_{\Sigma} = \sum_{i=1}^7 \Delta h_i$$

$$\Delta S_{2-3} = C_p \cdot \ln\left(\frac{T_3}{T_2}\right) - R \cdot \ln\left(\frac{P_3}{P_2}\right)$$

$$\Delta S_{6-0} = C_p \cdot \ln\left(\frac{T_0}{T_5}\right) - R \cdot \ln\left(\frac{P_0}{P_5}\right)$$

$$\Delta S_{\Sigma} = \sum_{i=1}^6 \Delta S_i$$

3.2 Calculation of heat energy

$$q_{0-1} = q_{1-2} = 0;$$

$$q_{3-4} = q_{4-5} = 0;$$

$$q_{2-3} = q_1 = C_p \cdot (T_3 - T_2)$$

$$q_{5-0} = q_2 = C_p \cdot (T_5 - T_0)$$

$$q_u = q_1 - q_2.$$

3.3 Calculation of work

$$-l_{0-1} = \Delta h_{0-1} \Rightarrow l_{0-1} = -\Delta h_{0-1} - \text{fan compression work}$$

$$-l_{1-2} = \Delta h_{1-2} \Rightarrow l_{1-2} = -\Delta h_{1-2} - \text{compressor work}$$

$$-l_{3-4} = \Delta h_{3-4} \Rightarrow l_{3-4} = -\Delta h_{3-4} - \text{turbine work}$$

$$-l_{4-5} = \Delta h_{4-5} \Rightarrow l_{4-5} = -\Delta h_{4-5} - \text{fan drive work}$$

$$-l_{5-6} = \Delta h_{5-6} \Rightarrow l_{5-6} = -\Delta h_{5-6} - \text{nozzle work}$$

$$l_{ll} = \sum_{i=1}^5 l_i \frac{J}{kg}.$$

4. Evaluating working medium properties at the passing points of compression and expansion processes

4.1 Evaluating passing point parameters at P-V GTE cycle diagram

Defining p and v at passing points helps in building accurate diagrams. As 1-2 and 3-4-5-6 processes are adiabatic, any two plot points obey the following:

$$\frac{p_a}{p_1} = \left(\frac{v_1}{v_a}\right)^{k'}; \frac{p_b}{p_1} = \left(\frac{v_1}{v_b}\right)^{k'}; \frac{p_c}{p_4} = \left(\frac{v_4}{v_c}\right)^{k''}; \frac{p_d}{p_4} = \left(\frac{v_4}{v_d}\right)^{k''}.$$

With usage of values p_1, p_6, v_1, v_6 , parameters of two passing points can be found. These parameters should be used in building cycle diagram.

4.2 Evaluating process parameters of T-S cycle

In order to build T-S cycle, temperature variation intervals should be subdivided into three approximately equal parts:

$$I.1) \Delta S_{2-a'} = c_p \ln \frac{T_{a'}}{T_2}$$

$$2) \Delta S_{2-b'} = c_p \ln \frac{T_{b'}}{T_2}$$

$$II.1) \Delta S_{5-c'} = c_p \ln \frac{T_{c'}}{T_5};$$

$$2) \Delta S_{5-d'} = c_p \ln \frac{T_{d'}}{T_5};$$

T-S cycle diagram can be built with usage of the values of temperature and entropy change found above (fig. 4).

5. Building p-v and T-S diagrams

Table 5. Parameter values at passing points

Parameter	Point			
	a	b	c	d
p_i, Pa				
$v_i, \text{m}^3/\text{kg}$				
Parameter	Point			
	a	b	c	d
T_i, K				
	Process			
	2- a'	2-b'	0-c'	0-d'
$\Delta S_i, \text{J/kg} \cdot \text{K}$				

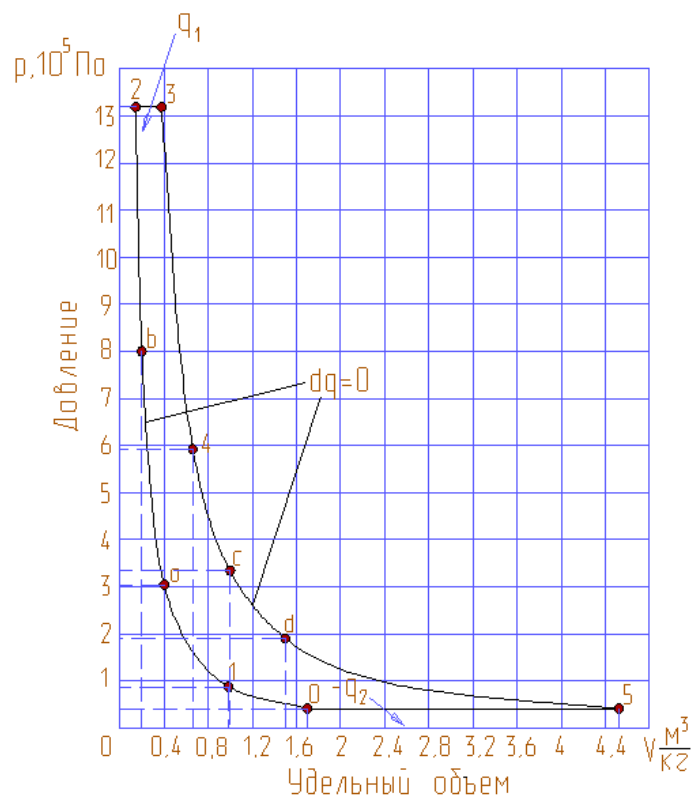


Fig. 3. p-v GTE cycle diagram

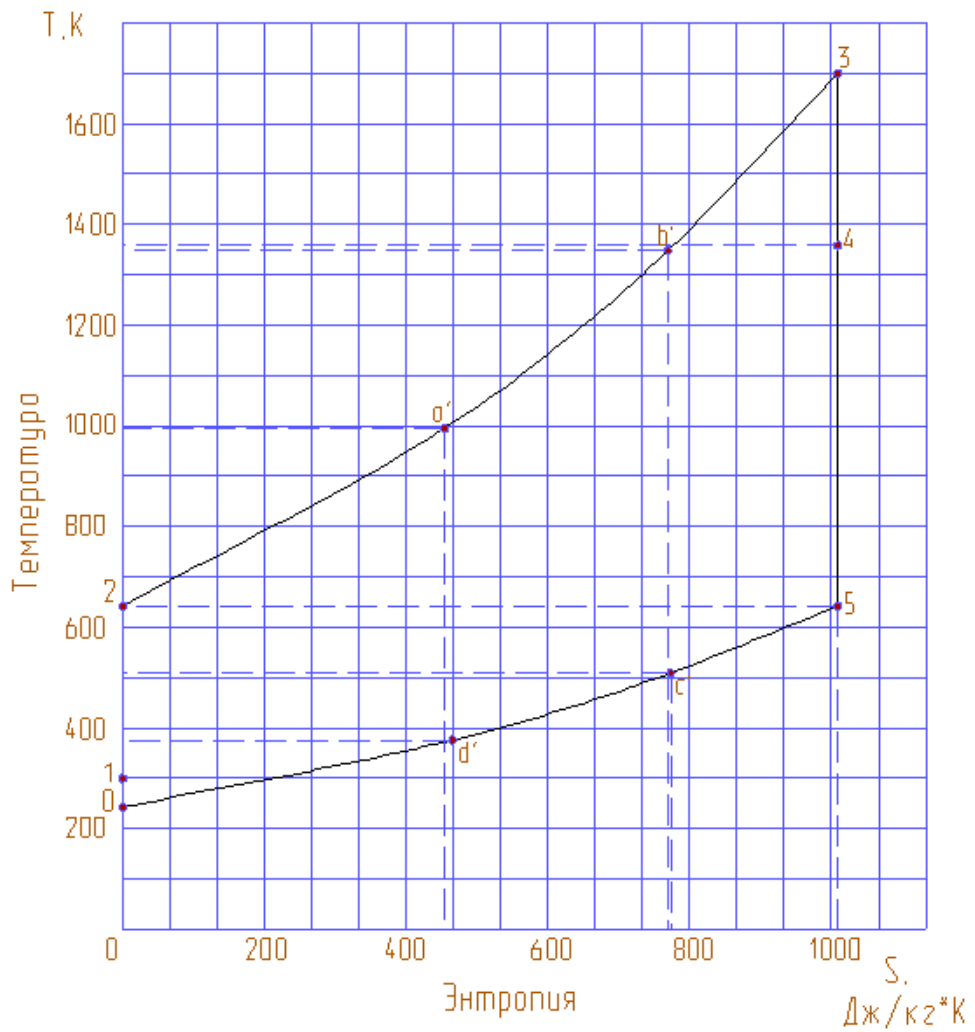


Fig. 4. T-S GTE cycle diagram

6. Evaluating energy characteristics of GTE

1) Working medium exhaust rate:

$$c_5 = \sqrt{2(h_4 - h_5)} = \sqrt{2 \cdot c_p \cdot (T_4 - T_5)};$$

2) Engine specific thrust:

$$R_{y\partial} = (c_5 - c_0) + \frac{m(p_1 - p_0)}{\rho_0 c_0};$$

3) Cycle thermal efficiency:

$$\eta_t = \frac{l_u}{q_1} \cdot 100\%;$$

4) Carnot cycle thermal efficiency:

$$\eta_t^k = \left(1 - \frac{T_0}{T_3}\right) \cdot 100\%.$$

Table 6. Energy characteristics of GTE

$(\pi_k)_{opt}$	$l_u, \frac{J}{kg}$	\square	$C_0, m/s$	$C_5, m/s$	
		$\eta_t, \%$	$\eta_t^k, \%$	$G_{возд}, kg/s$	$R_{y\partial}, m/s$

7. Evaluating cycle work by graphic method with usage of ADEM 3.0 program



Fig. 5. Evaluating p-v cycle area
 $l_{ц.гр.} = S_{ц} \cdot \mu_l$, here $S_{ц}$ – p-v cycle area;

$$S_{ц} = 1712,82 \text{ mm}^2;$$

$$\mu_l = 4 \cdot 10^2 \text{ J/kg} \cdot \text{mm}^2 \text{— scale};$$

$$l_{ц.гр.} = 1712,82 \cdot 4 \cdot 10^2 = 685128 \text{ J / kg}.$$

Calculation error:

$$\delta_1 = \frac{l_{ц.расч.} - l_{ц.гр.}}{l_{ц.расч.}} \cdot 100\%$$

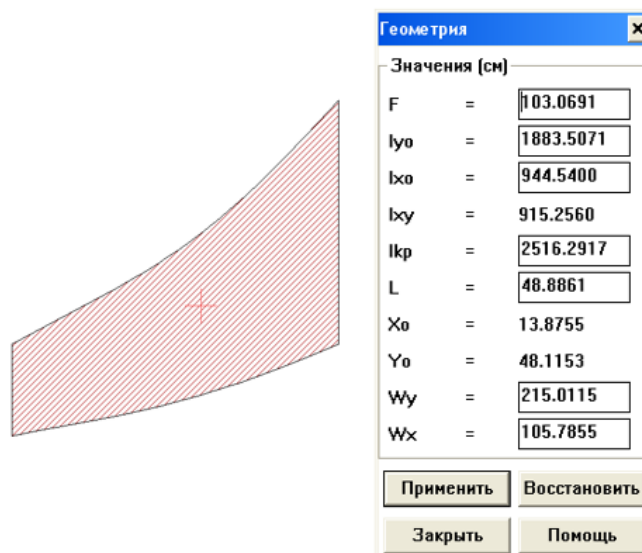


Fig. 6. Evaluating T-S cycle area
 $q_{ц.гр.} = S_{ц} \cdot \mu_q$, here $S_{ц}$ – T-S cycle area;

$$S_{ц} = 10306,91 \text{ mm}^2;$$

$$\mu_q = 66,7 \text{ J/kg} \cdot \text{mm}^2 \text{— scale};$$

$$q_{\text{ц.гр.}} = 10306,91 \cdot 66,7 = 687471 \frac{J}{kg}.$$

Calculation error:

$$\delta_2 = \frac{|q_{\text{ц.расч.}} - q_{\text{ц.гр.}}|}{q_{\text{ц.расч.}}} \cdot 100\%$$

Conclusion

This work deals with evaluation of thermodynamic GTE parameters: composition of working fluid, caloric and energy characteristics). Flight altitude, time, velocity, engine thrust and fuel sort are given as initial values.

T-S and p-v diagrams of GTE cycle are built.

For chosen temperature intervals, cycle thermal efficiency is lower than Carnot cycle thermal efficiency.