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SAMARA STATE AEROSPACE UNIVERSITY

Physics of the near Earth space

Electronic Laboratory Course

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Space physics practical works

Application to electronic practical works
Spacephysics
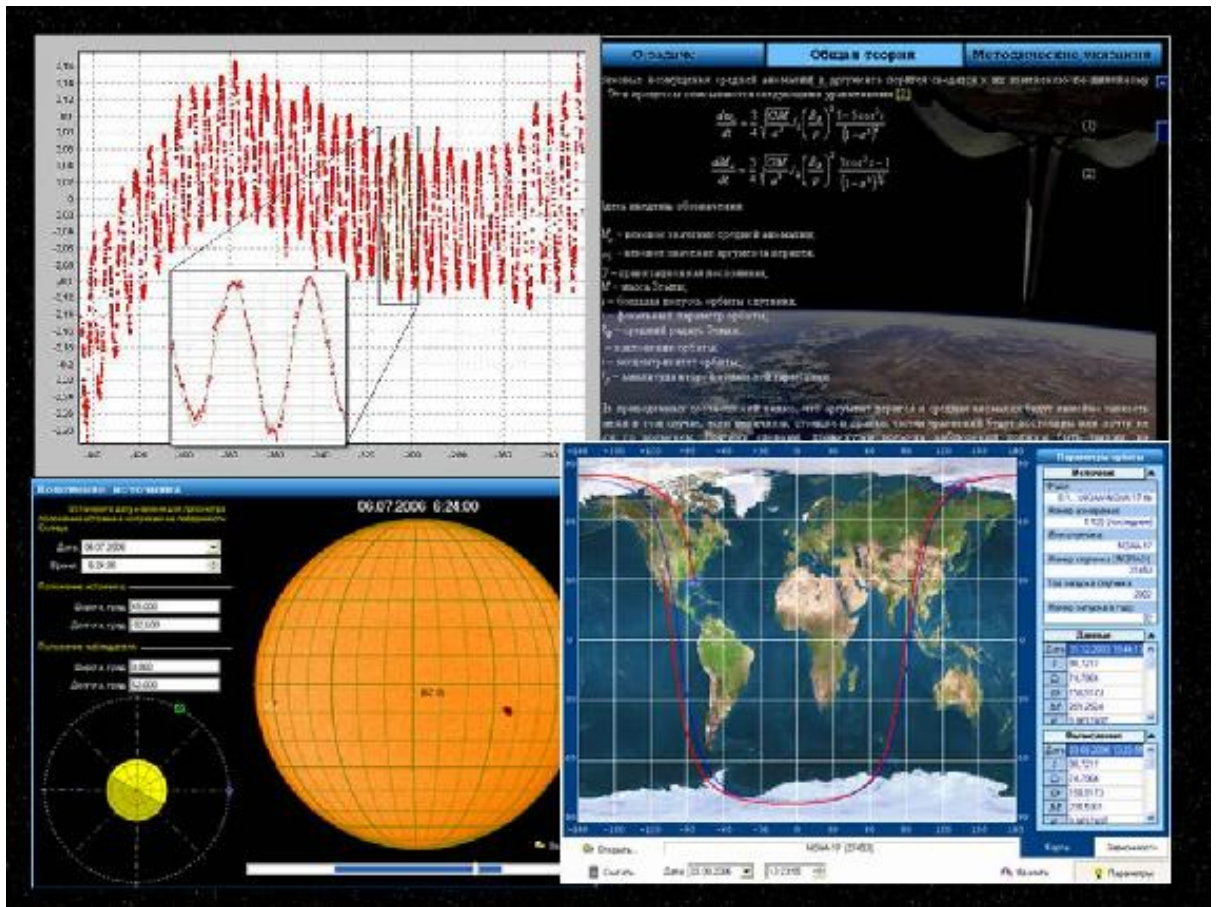
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Introduction into practical training

Unlike problems of general physical practical training where the most important role is played by problems in which a student must learn to work with different instruments he has at the moment, the cosmophysical problems use data received from instruments placed onboard spacecraft (SC) distantly or from satellite tracking services. Remote measurements represent one of the most widespread ways of physical experiment realization. This method is related not only to measurements on the orbit of onboard spacecraft but also to the conditions of realization of experiments inside active areas of nuclear reactors and chemical reactors, accelerators, deep-diving vehicles, medical experiments etc. Therefore skills and ability to work with the data received distantly make one of the most important components of a contemporary researcher training, and especially of a physicist.

Chapter 1.   The Earth. Motion of artificial satellites

1.1   The theoretical background

1.1.1   Motion of a point mass in the terrestrial gravitational field

Keplerian orbits

Newton's equations for a point particle with arbitrary mass in the gravitational field have the following form:

$$\frac{dv}{dt} = -\nabla\phi. \tag{1.1}$$

Here ϕ is gravitational potential of a field. Potential of the gravitational field of a spherically

symmetrical body has the form of:

$$\phi = \frac{GM}{r}.$$

In such field angular momentum of a satellite is constant:

$$L = [r \times p] = const.$$

Thereby it is convenient to choose coordinate system so that the plane of satellite's motion is orthogonal to the constant vector L. Let's direct axis z along vector L. in this case $L = (0,0,L_z)$. Changing to polar coordinate system in orbit plane we obtain:
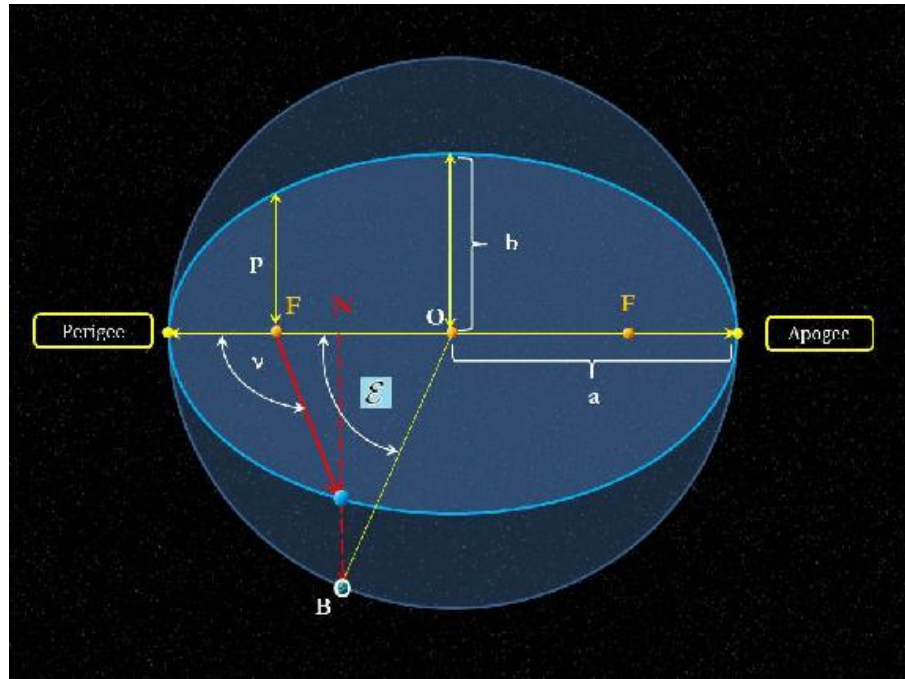
$$L_z = mr^2\dot{\phi} = const. \tag{1.2}$$

Fig. II-7-1. Elements of elliptic orbit

In chosen polar coordinates energy conservation law can be written like that:

$$E_0 = \frac{m}{2}(v_r^2 + r^2\dot{\phi}^2) - \frac{mMG}{r} = const. \tag{1.3}$$

Here v_r - radial velocity of a satellite, E_0 is its total energy. With (1.2) taken into account this conservation law takes the form which contains only radial coordinate functions:

$$E_0 = \frac{m}{2}\left(\left[\frac{dr}{dt}\right]^2 + \frac{L_z^2}{m^2r^2}\right) - \frac{mMG}{r} = const. \tag{1.4}$$

This conservation law form corresponds to one-dimensional radial movement of a particle in the potential gravitational field with the effective potential energy:

$$U_{eff} = \frac{L_z^2}{2mr^2} - \frac{mMG}{r},$$

which differ from potential energy of the initial problem by the presence of a summand corresponding to energy of orbital motion.

It is useful to transform (1.4) into the form of conservation law for harmonic oscillator to

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describe satellite's trajectory in examined gravitational field. It enables us to obtain series of relations based on this analogy to harmonic oscillator. Realizing sequentially substitution of variables $r(t) = r(\phi(t))$ and $\zeta(\phi) = r^{-1}(\phi)$ we obtain:

$$\frac{L_z^2}{2m}\left(\left[\frac{d\zeta}{d\phi}\right]^2 + \zeta^2\right) - mMG\zeta = E_0$$

And finally:

$$\left[\frac{d\zeta}{d\phi}\right]^2 + \left[\zeta - \frac{m^2 MG}{L_z^2}\right]^2 = \frac{2mE_0}{L_z^2} + \left[\frac{m^2 MG}{L_z^2}\right]^2 = const.$$

The obtained conservation law matches in a form with energy conservation law of a harmonic oscillator with unit mass, and equilibrium position and the position at a point with coordinate: $\zeta_0 = \dfrac{m^2 MG}{L_z^2}$. With that angular variable ϕ plays the role of time. As far the law of harmonic oscillator motion is known beforehand we can write the solution for its effective coordinate ζ like:

$$\zeta = \frac{1}{r} = \frac{m^2 MG}{L_z^2} + A\cos(\phi - \phi_0).$$

It is convenient to represent this solution in a form of ellipse canonical equation in polar coordinates:

$$r = \frac{p}{1 + e\cos v}, \tag{1.5}$$

where p is focal parameter, e is eccentricity. Angle $v = \phi - \phi_0$ counted from perigee direction is called the true anomaly. Perigee is a point of an orbit with the minimal distance from the field center, and apogee is a point with maximum distance (if such point exists). All these parameters are shown in fig. I-1. On this figure angle E relative to the center of ellipse counted from perigee to line OB is called eccentric anomaly E.

Using the obtained solution it is possible to link ellipse canonical parameters to dynamic parameters of satellite movement - its total energy and angular momentum. Some simple considerations are needed for this. Radial velocity component is equal to zero in apogee and perigee. Therefore we find from energy conservation law:

$$E_0 = \frac{L_z^2}{2mr_{a,p}^2} - \frac{mMG}{r_{a,p}},$$

where r_a and r_p are distances from the field center (the Earth) in apogee and perigee respectively. Summing these two relations for r_a and r_p and taking into account the definition of semimajor axis a: $a = (r_a + r_p)/2$ we obtain the following equation for the total energy:

$$E_0 = -\frac{mGM}{2a}. \tag{1.6}$$

Hence we obtain a useful relation for squared absolute velocity of orbital movement:

$$V^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right). \tag{1.7}$$

Now it is possible to obtain expressions for distances in apogee and perigee as functions of dynamic parameters of a satellite:

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$$\frac{1}{r_{a,p}} = \frac{m^2 MG}{L_z^2}\left(1 \pm \sqrt{1 + 2E_0 \frac{L_z^2}{M^2 m^3 G^2}}\right).$$

On the other hand it follows from ellipse canonical equation (1.5):

$$r_p = \frac{p}{1+e}, \quad r_a = \frac{p}{1-e}. \tag{1.8}$$

Hence we obtain expressions for impact parameter p and eccentricity e also as functions of dynamic parameters of satellite motion:

$$p = \frac{2r_p r_a}{r_p + r_a} = \frac{L_z^2}{m^2 MG}, \quad e = \frac{r_a - r_p}{r_a + r_p} = \sqrt{1 + 2E_0 \frac{L_z^2}{M^2 m^3 G^2}}. \tag{1.9}$$

Kepler equation is used to describe the motion of a satellite on the orbit. The equation has the form of:

$$E - e\sin E = n(t - t_0) + M_0. \tag{1.10}$$

Here E is eccentric anomaly, $M = n(t-t_0) + M_0$ is called mean anomaly, and M_0 **is mean anomaly at epoch** (t_0), $n = \sqrt{MGa^{-3}}$ is **mean motion**. The geometric sense of eccentric anomaly is clarified in fig. I-1. Eccentric anomaly is linked to natural anomaly (angle v in fig. I-1) by formula:

$$\frac{v}{2} = \sqrt{\frac{1+e}{1-e}}\frac{E}{2} \tag{1.11}$$

True anomaly is defined from these two equations, corresponding to the definite moment of time t and therefore satellite position on the orbit is defined for example relative to perigee.

Position of the orbit in space

Obtained relations allow calculation of parameters of elliptic orbit of a satellite. But it is necessary to know not only parameters of ellipse but also the position of orbit in space in relation to geocentric coordinate system. These geometric parameters of orbit position in geocentric rectangular coordinate system for Earth satellites are introduced in the following way. Z-axis of this coordinate system coincides with the Earth axis of rotation with positive direction to the North Pole. This point is situated near the Pole star on the celestial sphere. X-axis directs from the Earth center to the vernal equinox situated in the Golden Fish constellation of the modern epoch. Y-axis is perpendicular to the other two axes.
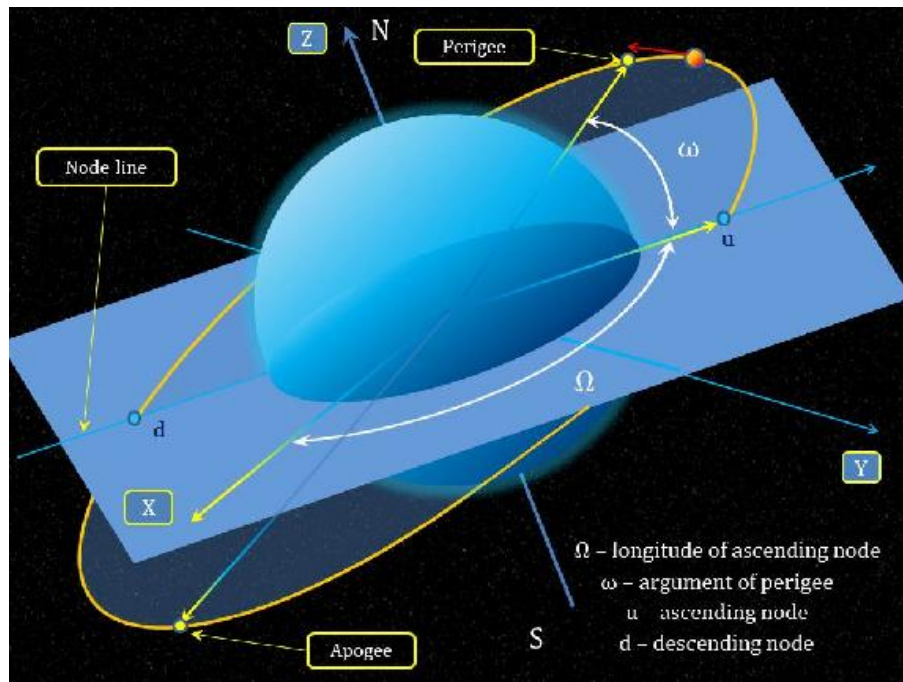
Fig. II-7-2. Position of the orbit in space.

The point where the orbit crosses the equatorial plane is called n**ode of orbit**. The point where a satellite passes from the southern hemisphere to the northern one is called **ascending node**. The point opposite to the Earth center is called **descending node**. Angle between the direction to the vernal equinox and ascending node, i.e. between the x-axis and ascending node, is called **longitude of ascending node** and is designated by Ω here. Angle in the plane of orbit counted from ascending node to perigee is called ascending node-perigee angle and is designated by ω. Angle between the plane of orbit and equatorial plane is called orbit inclination and is designated by i here. Orbit parameters also include focal parameter p and eccentricity e. These parameters are shown in fig. 2.

Cartesian coordinates of a satellite will have the following form in geocentric coordinate system:

$$x = r(\cos u \cos \Omega - \sin u \sin \Omega \cos i), \tag{1.12}$$

$$y = r(\cos u \sin \Omega + \sin u \cos \Omega \cos i), \tag{1.13}$$

$$z = r \sin u \sin i. \tag{1.14}$$

Here $u = v + \omega$. Details can be found in [1] [1]. The full set of formulae see in [4] (pages 171-180) [2].

Basic equations of the Keplerian orbit modeling

Suppose all Keplerian parameters of the orbit to be known, for example from NORAD data. To obtain image of the orbit in projection to the rotating Earth taking into account the satellite motion along the orbit it is useful to use the following equations for the preparatory calculation of the Cartesian coordinates in fixed coordinate system related to remote stars. These equations are [3]:

$$r = a(1 - e \cos E),$$

$$\xi = a(\cos E - e), \quad \eta = a\sqrt{1-e^2} \sin E,$$

[1] A.N. Matveev. Mechanics and the theory of relativity. M "Vysshaya shkola" 1986

[2] Handbook of celestial mechanics and astridynamics. Edited by Duboshin, M: "Nauka" 1971, 584

[3] Handbook of celestial mechanics and astridynamics. Edited by Duboshin, M: "Nauka" 1971, 584

$$x = P_x \xi + Q_x \eta, \tag{1.15}$$
$$y = P_y \xi + Q_y \eta,$$
$$z = P_z \xi + Q_z \eta,$$

where ξ and η are auxiliary orbital coordinates and direction cosine $P_x, P_y, \ldots, Q_z$ are calculated by:

$$P_x = \cos\Omega\cos(\omega) - \sin\Omega\sin(\omega)\cos i,$$
$$P_y = \sin\Omega\cos(\omega) + \cos\Omega\sin(\omega)\cos i,$$
$$P_z = \sin\omega\sin i,$$
$$Q_x = -\cos\Omega\sin(\omega) - \sin\Omega\cos(\omega)\cos i,$$
$$Q_y = -\sin\Omega\sin(\omega) + \cos\Omega\cos(\omega)\cos i,$$
$$Q_z = \cos\omega\sin i.$$

Direction cosines must satisfy the following conditions serving to control the correctness of calculations:

$$P_x^2 + P_y^2 + P_z^2 = 1,$$
$$Q_x^2 + Q_y^2 + Q_z^2 = 1,$$
$$Q_x P_x + Q_y P_y + Q_z P_z = 0.$$

It is clear that quantities $P_x, P_y, \ldots, Q_z$ are calculated once when the information about the new orbital parameters of satellite comes from ground-based services as they contain only Keplerian orbit parameters.

To calculate the motion of a satellite along the orbit in the projection on the celestial sphere it is necessary to use equations of transformation of Cartesian ordinate into spherical coordinates:

$$\lambda = \lambda_0 + \frac{y}{x}, \quad \theta = \arccos\frac{z}{r}, \tag{1.16}$$

where λ is the longitude of a projection point of a satellite's position on the celestial sphere, λ_0 - present longitude of the vernal equinoctial point where the X-axis of geostationary coordinate system is directed to, θ its latitude and the Cartesian coordinates x, y, z and the distance r from the Earth center to the satellite are calculated by the described equations.

The present position of a satellite in fixed coordinate system is calculated with the help of Kepler equation with respect to eccentricity anomaly E:

$$E - e\sin E = n(t - t_0) + M_0 = M, \tag{1.17}$$

where E is eccentricity anomaly, n - mean motion, t_0 - epoch, M_0 - mean anomaly in epoch. The numerical solution of this equation can be obtained with the help of iteration method:

$$E_{k+1} = e\sin E_k + n(t - t_0) + M_0, k = 0, \ldots.$$

The initial value should be chosen:

$$E_0 = n(t - t_0) + M_0,$$

And the process of iterations should be finished when needed accuracy is reached, i.e. when the quantity $E_{k+1} - E_k$ becomes less than a given value. For eccentricity $e < 0.5$ the number of iterations does not exceed 5 with the accuracy of 7-8 decimal positions.

Transition to the coordinate system rotating with the Earth is realized by the calculation of the longitude λ_0 from the equation:

$$\lambda_0(t) = \lambda_0(0) - \left(\frac{1}{365.242} + 1\right)\frac{(t-t_0)}{86400}360^o$$

where $\lambda_0(0)$ the value of longitude of vernal equinox point in epoch t_0. As far as the data concerning the time coming from NORAD is measured in days it is convenient to adduce these equations to these time units and angular variables to degrees which correspond to geographical coordinates.

The speed of satellite's motion along the orbit can be calculated in the following way. Differentiate (1.17) with respect to time. As the result we obtain:

$$\frac{dr}{dt} = V_r = -ae\frac{dE}{dt}, \quad \frac{d\xi}{dt} = -a\sin E\frac{dE}{dt}, \quad \frac{d\eta}{dt} = a\sqrt{1-e^2}\cos E\frac{dE}{dt},$$

$$V_\alpha = \frac{dx_\alpha}{dt} = P_\alpha\frac{d\xi}{dt} + Q_\alpha\frac{d\eta}{dt} = a(-P_\alpha\sin E + Q_\alpha\sqrt{1-e^2}\cos E)\frac{dE}{dt}, \quad \alpha = 1,2,3. \quad (1.18)$$

Differentiate (1.17) with respect to t, we find:

$$\frac{dE}{dt} = \frac{n}{1-e\cos E}.$$

Finally we find:

$$V_\alpha = \frac{dx_\alpha}{dt} = P_\alpha\frac{d\xi}{dt} + Q_\alpha\frac{d\eta}{dt} = an\frac{(-P_\alpha\sin E + Q_\alpha\sqrt{1-e^2}\cos E)}{1-e\cos E}, \quad \alpha = 1,2,3. \quad (1.19)$$

For a circular orbit $e = 0$

$$\frac{dE}{dt} = n = \text{const.}$$

In this case

$$V_\alpha = \frac{dx_\alpha}{dt} = P_\alpha\frac{d\xi}{dt} + Q_\alpha\frac{d\eta}{dt} = -anP_\alpha\sin(n(t-t_0)+M_0) + anQ_\alpha\cos(n(t-t_0)+M_0),$$

With the help of simple calculations we find from these relations [4]:

$$V^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right),$$

$$V_r = \sqrt{\frac{\mu}{p}}e\sin v,$$

$$V_n = \sqrt{\frac{\mu}{p}}(1+e\cos v).$$

And respectfully:

$$\dot{x} = \frac{x}{r}V_r + (-\cos\Omega\sin(\omega) - \sin\Omega\cos(\omega)\cos i)V_n = \frac{x}{r}V_r + Q_xV_n,$$

$$\dot{y} = \frac{y}{r}V_r + (-\sin\Omega\sin(\omega) + \cos\Omega\cos(\omega)\cos i)V_n = \frac{y}{r}V_r + Q_yV_n,$$

$$\dot{z} = \frac{z}{r}V_r + (\cos(\omega)\sin i)V_n = \frac{z}{r}V_r + Q_zV_n.$$

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[4]Handbook of celestial mechanics and astridynamics. Edited by Duboshin, M: "Nauka" 1971, p. 174

Motion in spheroidal field. Model of precession

In case if the gravitational field of a planet is not perfectly spherical the shape of the orbit and the character of the motion of a satellite along the orbit become more complicated. In the simplest case if the second zonal harmonic J2 is taken into account in the terrestrial gravitational field the gravitational field potential takes the following form:

$$U = -\frac{GM_E m}{r}\left[1 + \frac{J_2}{2}\left(3\frac{z^2}{r^2} - 1\right)\cdot\frac{R_E^2}{r^2}\right],$$ (1.20)

where m is mass of satellite and the equation $\cos\theta = z/r$ was used. Consideration of only zonal harmonics is equivalent to the supposition that the shape of the Earth is a solid of revolution around the axis passing through the poles of the planet. Denote this axis as Z. the influence of an additional summand leads to a slow rotation of the orbit plane around Z-axis. Together with the rotation of the orbit plane seen in the monotone increase of the ascending node longitude the position of the perigee also changes. So all quantities Ω, ω and M change in the course of time. The speed of change of those quantities is proportional to J_2. Therefore measuring quantities $\dot\Omega$, $\dot\omega$ and M_0 it is possible to calculate J_2 and estimate the degree of deviation of the shape of the Earth from sphere. For this purpose it is necessary to use equation:

$$J_2 = \frac{1}{5}\left[\left(\frac{R_P}{R_E}\right)^2 - 1\right],$$

Where R_P is polar radius of the Earth and R_E - equatorial radius of the Earth.

The more precise analysis considering possible deviation of an orbit from circular one shows that at a first approximation ascending node-perigee angle ω and mean anomaly M_0 also slowly change together with longitude of ascending node. Corresponding formulae for the rate of secular variation rate have the following form:

$$\frac{d\Omega}{dt} = \frac{3}{2}\sqrt{\frac{GM}{a^3}}J_2\left(\frac{R_E}{p}\right)^2\frac{\cos i}{(1-e^2)^2},$$ (1.21)

$$\frac{d\omega}{dt} = \frac{3}{4}\sqrt{\frac{GM}{a^3}}J_2\left(\frac{R_E}{p}\right)^2\frac{(1-5\cos^2 i)}{(1-e^2)^2},$$ (1.22)

$$\frac{dM_0}{dt} = \frac{3}{4}\sqrt{\frac{GM}{a^3}}J_2\left(\frac{R_E}{p}\right)^2\frac{(3\cos^2 i-1)}{(1-e^2)^{3/2}},$$ (1.23)

here $p = a(1-e^2)$ is orbit focal parameter. R_E is equatorial radius of the Earth, a - is semimajor axis. It is clear that for a circular orbit formula for Ω exactly coincides with the equation obtained earlier. These equations can be interpreted as a magnitude of orbit components changes in radian or angle measurement within some period of time, for example within on revolution of a satellite around the Earth. But as it will be shown further in real data parameters are usually given within twenty-four hours. Quantity $n = \sqrt{GMa^{-3}}$ is called mean motion and is given in NORAD data in a form of revolution number in twenty-four hours. Longitude of ascending node and ascending node-perigee angle is given in degree scale in NORAD data. In these units $\Delta\Omega$ and $\Delta\omega$ in twenty-four hours will be expressed in a following way:

$$\Delta\Omega_{day} = \frac{3}{2}360nJ_2\left(\frac{R_E}{p}\right)^2\cos i,$$ (1.24)

$$\Delta\omega_{day} = \frac{3}{4}360nJ_2\left(\frac{R_E}{p}\right)^2(1-5\cos^2 i),\tag{1.25}$$

$$\Delta M_{0day} = \frac{3}{4}360nJ_2\left(\frac{R_E}{p}\right)^2(3\cos^2 i - 1)\tag{1.26}$$

Quantity $360n$ represents the total angular incursion on the true anomaly within twenty-four hours. Now we can easily estimate magnitude of changes of longitude of ascending node, ascending node-perigee angle and mean anomaly within twenty-four hours for some typical satellites. So for polar meteorological satellites of NOAA type $n \sim 14\,rev/day$. $R_E/p \sim 1.2$, $i = 98^o$, $\cos i \sim -1.2^o$. Using value J_2 from Tab III-1 we obtain $\Delta\Omega_{day} \sim 1^o$, $\Delta\omega_{day} \sim 3^o$.
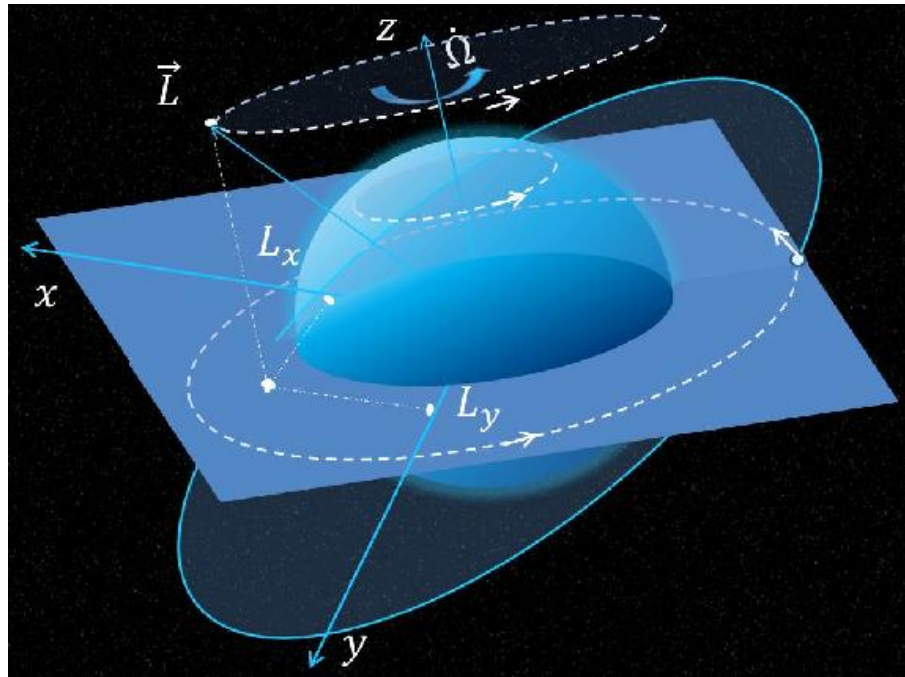

Fig. II-7-3. Rotation of orbit plane in space.

Formulae (1.24) and (1.25) are taken from [3][5]. Formula (1.26) is taken from reference guide [4](page 438) [6] containing reference information concerning all parameters of satellite motion. There is a slight difference between formulae given in [3] and [4]. The difference is in absence and presence (respectively) of multiplier $(1-e^2)^{-2}$ for the first two formulae and multiplier $(1-e^2)^{-3/2}$ for mean anomaly. These multipliers practically do not differ from 1 for nearly circular orbits. Details of theory of satellite motion in spheroidal field of the Earth can be found in [4] [7].

1.1.2 Atmospheric resistance influence on satellite motion

This practical task is targeted at the verification of a simple model of the effect of the force of molecular resistance form atmosphere side on satellites which move along relatively low and

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[5]V.V.Beletckiy. Essay about celestial mechanics. M:``Nauka'',1972, 320 p.

[6]Handbook of celestial mechanics and astridynamics. Edited by Duboshin, M: "Nauka" 1971, 584

[7]G.N. Duboshin. Celestial mechanics. Basic problems and methods. M.: Nauka, 1968, P.IV, Ch.XII, pp. 566-652

near-circular orbits.

Suppose satellite with mass m and cross-sectional area σ is moving along an orbit and experiencing frontal resistance of the atmosphere. Designate the concentration of molecules of atmospheric gases by N. Then the satellite collides with $dN = NdV$ molecules enclosed in volume $dV = v\sigma dt$ within time dt, so $dN = Nv\sigma dt$ (see fig. 6).
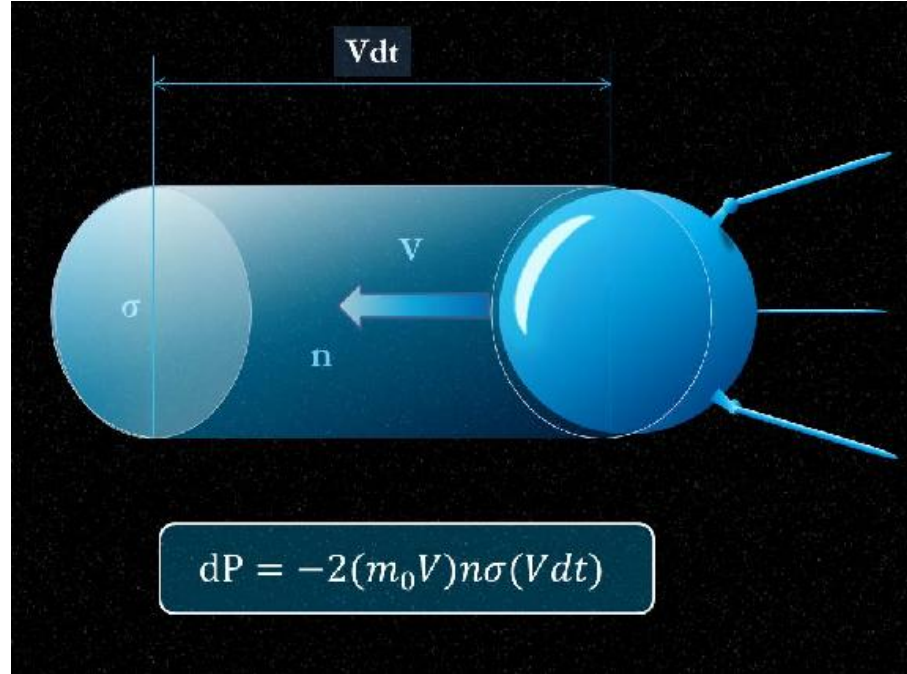

Fig. III-7-4 For molecular resistance force derivation.

Each impact is considered to be perfectly elastic. Then within time dt molecules give to the satellite momentum equal to $dp = 2m_0 v dN = 2v^2 m_0 \sigma N dt$. Here m_0 is mass of one molecule. Hence we obtain the resistance force experienced by a satellite:

$$\frac{dp}{dt} = F_d = -2v^2 \rho \sigma = -\gamma v^2,$$

where $\gamma = 2\rho\sigma$ is coefficient of friction, $\rho = m_0 N$ is density of the atmosphere.

Consider a satellite motion in the potential field of force under such friction force. Equation of motion of a satellite in Cartesian coordinate system has the following form:

$$m\frac{dv}{dt} = -\nabla U - F = -\nabla U - vv\gamma.$$

Then we multiply this equation scalarly by velocity of satellite's motion. After simple transformations we obtain:

$$\frac{d}{dt}\left[\frac{mv^2}{2} + U\right] = -\gamma v^3.$$

Hence total specific energy of a satellite $E_m = E_0/m$ changes according to the equation:

$$\frac{dE_m}{dt} = -\frac{\gamma}{m}v^3.$$

Now suppose a satellite to move along an orbit with small eccentricity $e \ll 1$, i.e. near-circular orbit.

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In this case radial component of satellite velocity is considerably less than the velocity of orbital motion $v_\phi = r\dot\phi$. So it is possible to consider:

$$v = \sqrt{v_r^2 + v_\phi^2} \simeq v_\phi = r\dot\phi = \frac{L}{rm} = \frac{L_m}{r},$$

where L - absolute magnitude of orbital angular momentum of a satellite, $L_m = L/m$ is specific angular moment. Then we obtain the following relation:

$$\frac{r^3}{L_m^3}\frac{dE_m}{dt} = -\gamma/m = const. \qquad (1.27)$$

Taking into account (1.9), relation (1.27) can be represented as the following:

$$-\frac{p^3}{(\mu p)^{3/2}}\frac{d}{dt}\left[\frac{(1-e^2)\mu}{p}\right] = -\gamma/m$$

here $\mu = M_E G$. As far as it is supposed that satellites orbits examined in this task are close to circular (small eccentricity) then $r \sim p$.

In addition disturbing additions related to nonspherisity of the Earth are neglected in the formula for total energy due to their smallness. As a result the latter relation can be reduced to a form:

$$\rho = -\frac{m}{\sigma\sqrt{\mu}}\frac{d\sqrt{p}}{dt}. \qquad (1.28)$$

Thus this formula allows us to estimate related change of atmospheric density at a satellite's orbit height. It is necessary to know cross-section and mass of a satellite to calculate absolute value of atmospheric density.

1.2 Input data for practical work

Problems of cosmophysical practical training referred to section "Mechanics" request skills of work with data on Keplerian elements of satellite's orbit which comes from tracking stations. One of the most complete databases of satellite regularly enriched and which has an open access for the most satellites is NORAD database (North American Aerospace Defense Command). But there are also other databases on other sites. Needed data can be received by an open access from a private site DR. T.S.Kelso [8]: www.celestrak.com.

NORAD data in TLE format

Data received from the NORAD service in TLE format has a form of three-lines records. One of them is not obligatory and contains the name of a satellite (line 0). Two another obligatory lines contain the complete information about the Keplerian orbit of a satellite. Each line has the following pattern:

0	AAAAAAAAAAAAAAAAAAAAAAAAA
1	nnnnnU nnnnnaaa nnnnn.nnnnnnnn +.nnnnnnnn +nnnnn-n +nnnnn-n nnnnnn
2	nnnnn nnn.nnnn nnn.nnnn nnnnnnn nnn.nnnn nnn.nnnn nn.nnnnnnnnnnnnnnn

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[8]TS.Kelso@celestrak.com

Line 0 represents 24-symbol (24 bytes) name of a satellite assigned in NORAD system (NORAD SATCAT).

Lines 2 and 3 contain the information according to table 2 (pattern form should be considered).

Line 1	
Number	Description
byte	
01	Number of line
02	Space
03-07	Number assigned to a satellite in NORAD system
08	Satellite classification (U- not classified)
09	Space
10-11	International code (the last two digits of the launch year)
12-14	International code (launch number in the year)
15-17	International code (number of stage)
18	Space
19-20	Epoch year (the last two digits)
21-32	Epoch (day of the year and fraction of the day with decimal dot)
33	Space
34-43	The first time derivative of mean motion (with decimal dot)
44	Space
45-52	The second time derivative of mean motion (decimal dot is supposed)
53	Space
54-61	Drag coefficient in the atmosphere (BSTAR drag term)
62	Space
63	Ephemerid type
65-68	Number of elements
69	Checksum (modulo 10)
	(letters, spaces, dots, ``+"-sign - 0, ``-" sign - 1)

Line 2	
Number	Description
byte	
01	Line number
02	Space
03-07	Number assigned to a satellite in NORAD system
08	Space
09-016	Inclination (in degrees)
17	Space
18-25	longitude of ascending node (in degrees)
26	Space
27-33	Eccentricity (decimal dot is supposed)

34	Space
35-42	Perigee argument (in degrees)
43	Space
44-51	Mean anomaly (in degrees)
52	Space
53-63	Mean motion (revolutions per day)
64-68	Number of revolutions per epoch ([Revs])
69	Checksum (modulo 10)
	(letters, spaces, dots, ``+''-sign - 0, ``-'' sign - 1)

Table 2. data format in two-line element of Keplerian orbit description in NORAD system.

All the orbit data is referred to one epoch indicated in the first line. Remember that longitude of ascending node is counted from the direction to the vernal equinox point the direction of which is (for convenience) tied to fixed celestial sphere. This direction slowly moves due to shift of terrestrial rotation axis and other processes. But for the practical training tasks it is possible to consider with a high accuracy that equatorial plane, direction to the vernal equinox point and terrestrial rotation axis are fixed.
Example:

NOAA 14 1 23455U 94089A 97320.90946019 .00000140 00000-0 10191-3 0 2621 2 23455 99.0090 272.6745 0008546 223.1686 136.8816 14.11711747148495

In this fragment a meteorological satellite with name NOAA14 has number 23455 in NORAD system. The satellite is not defined - parameter U. The launch year - 1994. Launch number in that year - 89. Epoch, when orbit parameters were measured - 320.90946019 day in 1997, i.e. positions of rotation axis of the Earth, equatorial plane and vernal equinox direction refer to that moment. Then inclination of the orbit is 90.0090 degrees, longitude of ascending node in epoch is 272.6745. Eccentricity is 0.0008546 (the orbit is almost circular). Perigee argument is 223.1686 degrees. Mean anomaly in epoch 136.8816, mean motion - 14.11711747 or the number of revolutions by the moment of measurement is 14849
Files containing a set of such records usually have TLE extension and are used by many program complexes of satellite tracking and amateur programs (e.g. Orbitron (www.stoff.pl)). Realizing task with a computer in practical training it is useful to have a copy of data on set of satellites in TLE format for several years. Such data is for example on www.celestrak.com. A fragment of such file is represented in the table for satellite NOAA17:
1 27453U 02032A 03036.91173877 .00000252 00000-0 13090-3 0 3431
2 27453 98.7603 108.1893 0012457 36.6226 323.5801 14.23284986 32161
1 27453U 02032A 03037.12263737 .00000259 00000-0 13369-3 0 2941
2 27453 98.7603 108.3991 0012453 35.9336 324.2677 14.23285178 32198
1 27453U 02032A 03037.89593266 .00000255 00000-0 13208-3 0 2957
2 27453 98.7602 109.1685 0012437 33.8535 326.3432 14.23285612 32304
1 27453U 02032A 03037.96623227 .00000255 00000-0 13205-3 0 2975
2 27453 98.7602 109.2384 0012436 33.6520 326.5446 14.23285653 32315
1 27453U 02032A 03038.95042518 .00000275 00000-0 14086-3 0 3378
2 27453 98.7600 110.2175 0012397 30.9679 329.2227 14.23286491 32457
1 27453U 02032A 03039.09102410 .00000280 00000-0 14294-3 0 2963
2 27453 98.7600 110.3574 0012393 30.4998 329.6899 14.23286631 32473

```
1 27453U 02032A 03039.93461765 .00000289 00000-0 14705-3 0 2974
2 27453 98.7599 111.1966 0012352 28.3357 331.8490 14.23287295 32596
1 27453U 02032A 03040.91880989 .00000291 00000-0 14782-3 0 3437
2 27453 98.7597 112.1756 0012326 25.4619 334.7164 14.23287992 32732
1 27453U 02032A 03041.12970817 .00000294 00000-0 14934-3 0 2982
2 27453 98.7597 112.3855 0012317 24.8073 335.3700 14.23288166 32761
```

1.3   Tasks of the practical training

1.3.1   Task 1. Measurement of J2 by secular variations of Keplerian parameters of satellite's orbit

Measurement of parameter J_2 - amplitude of the second zonal harmonic - can be realized with the help of any equation   (1.24) - (1.26) . Curves of a secular variation of three basic parameters - longitude of ascending node, perigee argument and mean anomaly represent almost straight lines the inclination of which contains the information of the second zonal harmonic magnitude J_2. For NOAA-17 these curves are shown in fig. 7-9. There also a straight line of linear regression is shown with the red color. In fig. 10 a deviation of change of longitude of ascending node from secular one is shown. It is obtained by the subtraction of regression line from the initial data. This deviation represents harmonic oscillations of the longitude of ascending node and perigee argument in relatively short segments. Parameters of these oscillations can also be used for measurement of magnitude of the second zonal harmonic amplitude.

Task.

1. For several given satellites with different inclination of the orbit make a selection from NORAD data file based on satellite's orbit parameters in definite period of time. Selection should be made by the longitude of ascending node Ω, Mean anomaly M_0, perigee argument ω, mean motion n and inclination i.

2. Graph the variation of difference $\Delta\Omega = \Omega - \Omega_E$ of the longitude of ascending node Ω in the given period of time and the diurnal rotation of the Earth Ω_E for all satellites. In the similar way graph mean anomaly M_0 and perigee argument ω.

3. Calculate the average value of focal parameter p of the orbit by the data on mean motion n and estimate statistical error of calculation of p. Do the same for inclination of the orbit.

4. 4. Using this data estimate tangent of inclination $R_{anl} = \alpha$ of graphs of secular shift of longitude of ascending node, perigee argument and mean anomaly. Parameter R_{anl} in units used represents the shift value of longitude of ascending node in degrees per day. With the help of equations   (1.24) ,   (1.25) amd   (1.26)) magnitude of zonal harmonic J2 amplitude can be calculated J_2:

$$J_2 = \frac{2}{3} \frac{R_{anl}}{360 n \cos i} \left(\frac{p}{R_E} \right)^2,$$

$$J_2 = \frac{4}{3} \frac{R_{arp}}{360n(1-5\cos^2 i)} \left(\frac{p}{R_E}\right)^2,$$

$$J_2 = -\frac{4}{3} \frac{R_{aan}}{360n(3\cos^2 i - 1)} \left(\frac{p}{R_E}\right)^2.$$

Here n is taken from NORAD data, i.e. in units - number of revolutions per day. Notice that number $360n$ represents the total run of the angle (true anomaly) per day. Multiplier $(1-e^2)^2$ is neglected due to its proximity to 1.

     5. Estimate statistical error in J_2 estimation

     6. Compare results for several satellites.

Questions for the preparation for the solution of the problem

     1.   What is zonal, tesseral and sectoral spheric harmonics of the planet field? Describe qualitatively their basic parameters.

     2.   In what way are amplitudes of the second harmonics related to the terrestrial moment of inertia?

     3.   The values of the second harmonics for solar system planets are given in table 1. Give the qualitative explanation concerning the difference in the second and subsequent harmonics of different planets.

     4.   What is planetary node? Explain its geometrical sense.

     5.   Describe qualitatively the character of change of the longitude of ascending node for equatorial and polar orbits of a satellite.

     6.   What is osculating elements of the orbit?

     7.   What is perigee argument of a satellite? Explain its geometrical sense.

     8.   Describe qualitatively the character of change of perigee argument for equatorial and polar orbits of a satellite

     9.   What is critical inclination of the orbit?

     10.   What is true, eccentric and mean anomaly? explain their geometrical sense.

     11.   Write (derive) Keplerian equation. what is the relation between true, eccentric and mean anomaly?

1.3.2 Task 2. Measurement of J_2 by long-period variations of Keplerian parameters of satellite's orbit

Measurement of parameter J_2 - amplitude of the second zonal harmonic can be realized with the help of equations of long-period perturbations of the orbit which are related to J_2. Long-period perturbations for perigee argument and mean anomaly represent almost harmonic deviations from secular perturbations amplitude of which is proportional to the second zonal harmonic J_2. According to the theory of perturbations (see [4]) corrections for the secular perturbations have a form of:

$$M = M' + \frac{1}{16} J_2 \left(\frac{R_{Eth}}{a} \right)^2 \eta^3 [1 - 11\theta^2 - 40\theta^4 (1 - 5\theta^2)^{-1}] \sin 2\omega', \qquad (1.29)$$

$$\omega = \omega' - \frac{1}{16} J_2 \left(\frac{R_{Eth}}{a} \right)^2 [1 - 11\theta^2 - 40\theta^4 (1 - 5\theta^2)^{-1}] \sin 2\omega' + O(e_0^2).$$

Here the designation is introduced:

$$\eta = \sqrt{1 - e_0^2}, \ \theta = \cos(i_0).$$

In these equations parameters M', ω' coincide with the secular variation of the orbit accurate within J_2^2, i.e. grow linearly.

To calculate the corrections it is necessary to subtract secular perturbations M', ω' obtained in the first practical work from the graph of change of $M(\text{å}), \omega(\text{å})$. Residual series represents long-period perturbations of the orbit with contribution of other outer forces for example the gravitational force of the Moon and the Sun. but these quantities are relatively small. Amplitude of periodic change in the residual series can be estimated with the help of method of least squares for a probe harmonic function. The typical form of residual perturbations from the secular motion of orbital parameters for NOAA-17 is shown in fig. I-5.
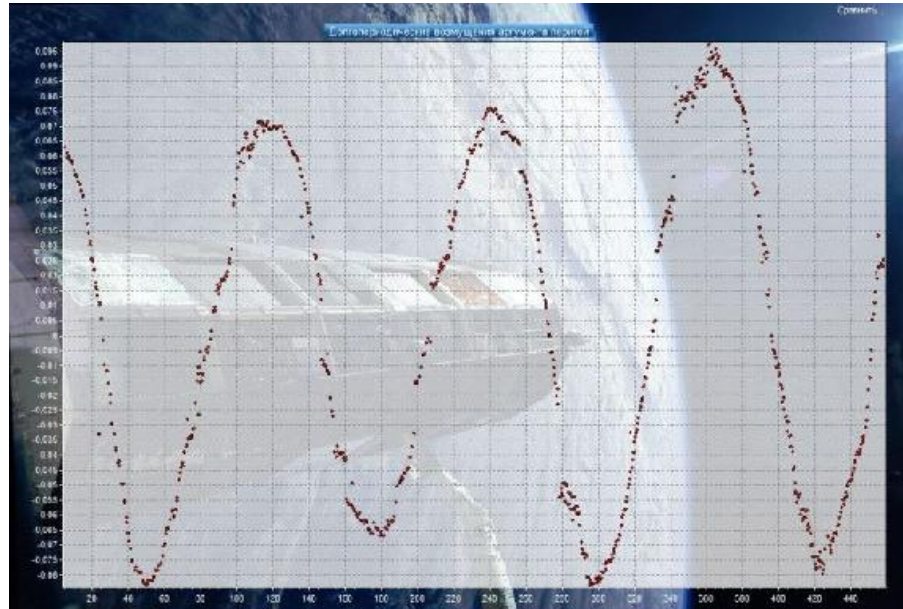

Fig. I-5. Residual series of perigee argument for satellite NOAA-17.

Task.

1. For several given satellites with different inclination of the orbit make a selection from NORAD data file based on satellite's orbit parameters in definite period of time. Selection should be made by mean anomaly M_0, perigee argument ω, mean motion n and inclination i.

2. With the help of method of least squares estimate tangent of inclination $R_{anl} = \alpha$ of graphs of secular shift of perigee argument and mean anomaly and points of shift of linear regression.

3. Basing on input data calculate the obtained linear regression dependences of $\omega'(t)$ and $M_{0'}(t)$.

4. With the help of method of least squares for a probe harmonic function calculate the amplitude of oscillations in the residual series. Using equations (1.29) estimate quantity J_2.

5. Estimate statistical error of J2 estimation J_2.

6. Compare the results for several satellites and compare results to the estimation obtained by secular perturbations.

Questions for the preparation for the solution of the problem

1. What are zonal, tesseral and sectoral harmonics of the planet's field? Describe qualitatively their basic characteristics.

2. In what way are amplitudes of the second harmonics related to the terrestrial moment of inertia?

3. Values of zonal harmonics for planets of the solar system are given in table 1. Give the qualitative explanation to relative differences in the second and subsequent harmonics of different planets.

4. What is orbital node? Describe qualitatively the character of change of the longitude of ascending node for equatorial and polar orbits of a satellite.

5. What is osculating elements of the orbit?

6. What is perigee argument of a satellite? Explain its geometrical sense.

7. Describe qualitatively the character of change of perigee argument for equatorial and polar orbits of a satellite

8. What is critical inclination of the orbit?

9. What is true, eccentric and mean anomaly? explain their geometrical sense.

10. Write (derive) Keplerian equation. what is the relation between true, eccentric and mean anomaly?

1.3.3 Task 3. Exploration of variation of atmospheric density at the satellite's height with the help of method of estimation of atmospheric resistance magnitude

Examining the variation of the mean motion within long periods of time from several months to tens of years it is possible to discover the changes of the atmospheric density within this period of time. Variation of atmospheric density at the height of satellite's orbit is caused by two basic reasons. The first one is a slow descent of the satellite's orbit and the growth of atmospheric density at lower

orbits. The second reason of atmospheric density variation is its warming by the solar radiation. The most interesting task in this problem is the disclosure of the way the atmospheric density changes due to its warming by the solar radiation.
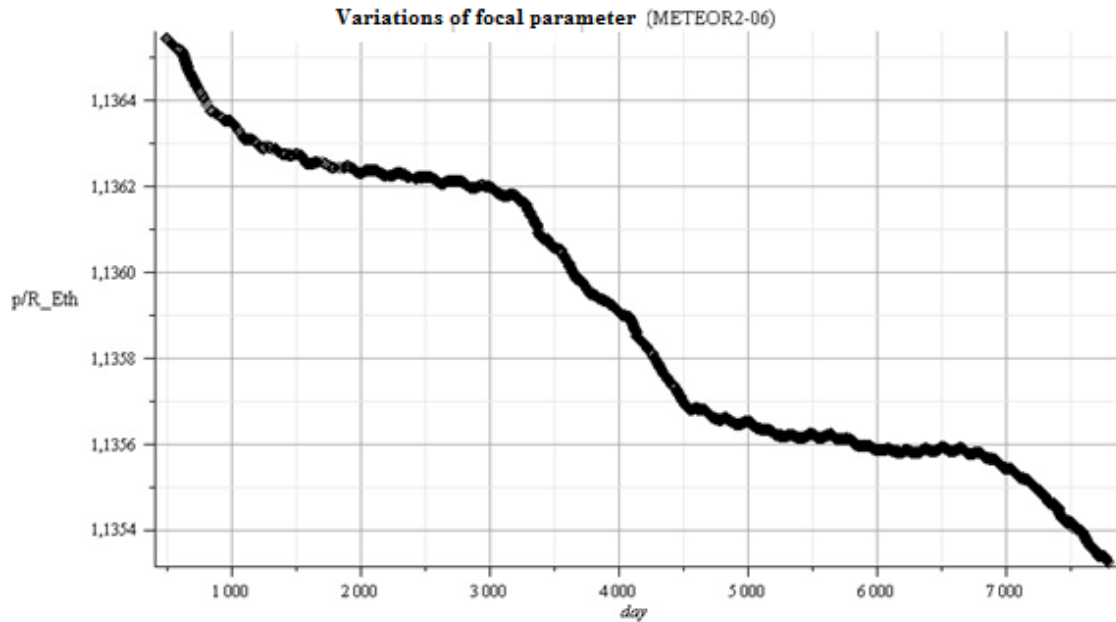

Fig. I-6. The example of calculation of the dependence of focal parameter on time.

There are two basic phenomena to be observed with the help of exploration of variation of satellite's orbit radius. The first one can be observed within periods of several months. As far as the atmosphere is constantly heated by the Sun on the side facing the Sun the atmosphere is more stretched to the Sun than in perpendicular direction. As the result when analyzing the data it is possible to observe the variation of density which have typical period coinciding with the period of secular rotation of the orbital plane of a satellite.

In order to complete this laboratory work it is necessary to select data concerning motion of satellite with near-circular orbits from the NORAD database. Satellites should be chosen that have orbits remaining unchanged within quiet a long period of time thanks to orbital maneuvering drives. Analysis of orbits can be verified with the examination of graphs of variation of longitude of ascending node, perigee argument ω, mean anomaly M_0, mean motion n and inclination i. After the selection of data one can start the work.

Tasks.

1. For satellites given by a teacher choose data on orbital elements into separate file with the help of a program written. (see exemplary listings in Maple.).

2. Graph the variation of the specific total energy, specific angular momentum and focal parameter.

3. With the help of method of smoothing of series, for example with the help of low-frequency filtration, calculate smoothed series of parameter $\sqrt{p(t)}$.

4. With the help of calculation of the central (right or left) difference derivative or with the help of the method of least squares calculate the speed of parameter $\sqrt{p(t)}$ variation as a time function in the whole time interval. Data obtained in tasks 3 and 4 should be plotted in one graph. Analyze graphs and explain it (additional information concerning the solar activity could be used).

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5. Supposing a satellite mass and cross-section area to be constant estimate the relative variation of atmospheric density at the height of satellite's orbit within the period of several months to several years. With the help of harmonic analysis calculate the typical period of atmospheric density variation and compare it to the angular velocity of the secular rotation of the orbital plane.

Questions for the preparation for the solution of the problem

1. In what way does the atmospheric resistance influence the shape of the orbit?
2. Describe qualitatively the variation of shape of a very elliptical orbit when a satellite passes near the Earth (in perigee)?
3. Describe qualitatively the variation of near-circular orbit parameters near the Earth due to the resistance of the atmosphere?
4. What can cause short-period growth of drag coefficient of the atmosphere?

1.3.4 Task 4. Exploration of interconnection between the atmospheric density variation and the solar activity

The second phenomenon observable with the help of estimations is related to the variation of solar activity with a typical period of 11 years. With the growth of solar activity the atmosphere of the Earth warms up much more due to increase of charged particles flows falling on the Earth. It leads to a considerable growth of speed of reduction of satellite's orbit radius. This phenomenon can be observed if one watches satellites having been on the orbit for quite a long time, i.e. several years or even more.

To complete this work it is necessary to choose orbital parameters for a long enough period of time for a satellite or satellites from NORAD database and to calculate values of specific total energy, specific angular momentum and semimajor axis and then to graph them. Then one should estimate the speed of decrease of total energy of a satellite with the help of least-squares method. And after that it is possible to calculate the specific drag coefficient of the atmosphere.

Task.

1. *For satellites given by a teacher choose data on orbital elements into separate file with the help of a program written. (see exemplary listings in Maple.).*

2. Graph the variation of the specific total energy, specific angular momentum and focal parameter.

3. With the help of method of smoothing of series, for example with the help of low-frequency filtration, calculate smoothed series of parameter $\sqrt{p(t)}$. Adjust the filter length in such way to leave in series measurements with a typical scale not less than one year.

4. With the help of calculation of the central (right or left) difference derivative or with the help of the method of least squares calculate the speed of parameter $\sqrt{p(t)}$ changing as a time function in the whole time interval. Data obtained in tasks 3 and 4 should be plotted in one graph. Analyse the graphs and explain it using the additional information about the solar activity. Data on series of Wolf number should be taken as such information (e.d. on a site: ftp://ftp.ngdc.noaa.gov/stp/solar_data/sunspot_numbers/international/monthly/MONTHLY.PLT. .

5. It is necessary to smooth the data on the atmospheric density to compare it to series of Wolf numbers. Overlay the graphs and calculate correlation coefficient between the relative density of the atmosphere and Wolf numbers.

1.3.5   Task 5. Modeling of satellite's motion along low orbits near the Earth

One of the problems one has to solve analyzing the data coming from satellites is calculation of the satellite's position relative to the Earth where any measurement of the space medium was carried out. It is necessary to solve this problem because the position of a satellite cannot always be obtained from a direct measurement of its coordinates. For many satellites their position can be detected only by ground-based tracking services which transmit the data from several times per day to several times per week. In all intermediate moments the satellite's position is calculated by the solution of the problem of satellite's motion.

In this problem a short program is supposed to be written to calculate the geographical coordinates of satellite's position projection on the celestial sphere at the given moment of time if Keplerian parameters of the satellite's orbit are known. It is supposed to be realized with nonsphericity of the Earth and other disturbing factors not taken into account.

In this task it is supposed to calculate the satellite's orbit using the data accessible from the NORAD system, such calculations can be carried out with the help of equations (1.15), (1.16), (1.17) etc. in this case we offer to explore orbits of satellites the orbital height of which is in the range from hundreds to several thousand kilometers.

Task.

1. For satellites given by a teacher choose data on orbital elements in some epoch t_0.

2. Using the chosen parameters graph the projection of satellite's orbit on the geographic map taking into account rotation of the Earth.

3. Compare the calculated position of a satellite to its real position respective to NORAD system data. Estimate the accuracy of coincidence of coordinates.

4. Explain deviations.

1.3.6   Task 6. Modeling of satellite's motion along geostationary orbit

Specific types of orbits lead to some interesting facts when projecting them on the Earth's surface. Geostationary orbits are the ones. There are a lot of scientific and telecommunication satellites on these orbits. That is why it is necessary to be able to calculate an orbit of a geostationary satellite.

Task.

1. For satellites given by a teacher choose data on orbital elements in some epoch t_0.

2. Using the chosen parameters graph he projection of satellite's orbit on the geographic map taking into account rotation of the Earth.

3. Compare the calculated position of a satellite to its real position respective to NORAD system data. Estimate the accuracy of coincidence of coordinates.

4. Explain the difference between the behaviors of different geostationary satellites.

1.4 Appendix 1. Texts of programs for realization of tasks

The text of a program in C (of Borland c company) allowing to chose needed data from files containing satellite data.

```c
#include <conio.h> #include <stdio.h>
#include <mem.h>
#include <string.h>
#include <math.h>
#include <stdlib.h>
void main(void)
{
// Variable type FILE for satellite data - file and
// graph - output files
FILE *file,*graph;
// String variable Buff using for reading
// line elements char
Buff[141];
// Set names for
files (FileName[60])
// file (FileGraph[60])
char FileName[60]="NOAA-17.txt";
char FileGraph[60]="ALNOAA17.txt";
int l,i;
// Var data for epoch.
// AscNodLong - longitude of ascending node (grad)
// amove - mean motion s^{-1}.
// p - focal parameter m
// a - big halfaxis m
// e - eccentrisity
// mu=M_Eth*G - Mass Earth non Grav. const G
// Energym - Energy per one kg mass
// Lm - specific angular momentum
double data,AscNodLong,amove,de2,p,e,a,mu,Energym,Lm;
double Pi=3.141592L,
// nu,ber Pi R_Eth=6378000,
// Equatorial radius of the Earth [m] M_Eth=5.98E24,
// Mass of the Earth [kg] G=6.672E-11;
// Gravitational constant in SI mu=M_Eth*G;
// String variable char SData[13];
// Epoch char SAMove[13];
// Mean motion (1/day) char SAscNodLong[10];
// longitude of ascending node (grad) char SEccentr[8];
// eccentrisuty *10^7 (mantissa without decimal point) char SIncl[9];
// Inclination (grad) char SArgPer[9];
// Argument peregee (grad) char Sp[9];
// Focal parameter
```

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```c
// Clean str var
setmem(Buff,141,0);
setmem(SData,13,0);
setmem(SAscNodLong,9,0);
setmem(SAMove,13,0);
setmem(SEccentr,8,0);
setmem(SIncl,9,0);
setmem(SArgPer,9,0);
setmem(Sp,9,0);
// Open files
if(NULL ==(file=fopen(FileName,"r") ))
{
puts("File error!"); exit(0);
}
if(NULL ==(graph=fopen(FileGraph,"w") ))
{
puts("File error!"); exit(0);
}
// Reading the file
for(i=1;i<100;i++)
{
// Copy to buffer
fread(Buff,1,140,file);
// Copy to var
memcpy(SData,&Buff[20],12);
memcpy(SData,&Buff[20],12);
memcpy(SAMove,&Buff[70+52],11);
memcpy(SEccentr,&Buff[70+26],7);
memcpy(SIncl,&Buff[70+8],8);
memcpy(SArgPer,&Buff[70+34],8);
memcpy(SAscNodLong,&Buff[70+17],8);
// Calculation of var
data=atof(SData);
amove=atof(SAMove)*2.0L*Pi/(3600.0L*24.0L);
a=pow(mu/(amove*amove),1.0L/3.0L);
e=atof(SEccentr)/10000000.0L;
de2=(1-e*e);
p=de2*a/R_Eth;
Energym=-de2*mu/p; Lm=sqrt(mu*p);
AscNodLong=atof(SAscNodLong);
setmem(Sp,9,0);
sprintf(Sp," setmem(SEccentr,8,0);
// Writing to output files
fprintf(graph," SData,SAscNodLong,SIncl,Sp,SArgPer,SEccentr,SAMove,Energym,Lm);
}
 // Close file
fclose(file); fclose(graph);
 }
```

Example of output file
175.81115570 242.8893 98.7855 1.12925 5.7429 0.00149 14.21021993 -3.533192e+14 2.122624e+07
175.84076954 242.9161 98.7900 1.12831 333.1378 0.00101 14.22801375 -3.536141e+14 2.121740e+07
175.88146355 242.9592 98.7879 1.12813 331.0799 0.00108 14.23131149 -3.536687e+14 2.121576e+07
175.89388638 242.9710 98.7876 1.12812 330.2629 0.00106 14.23159387 -3.536734e+14 2.121562e+07
175.89512742 242.9721 98.7873 1.12811 330.2535 0.00105 14.23166583 -3.536746e+14 2.121559e+07
175.91822197 242.9955 98.7878 1.12811 330.2151 0.00111 14.23177189 -3.536763e+14 2.121553e+07
175.97060955 243.0459 98.7876 1.12814 331.7044 0.00108 14.23118277 -3.536666e+14 2.121583e+07
176.10887502 243.1851 98.7874 1.12816 331.9446 0.00114 14.23079723 -3.536602e+14 2.121602e+07
176.51425062 243.5873 98.7867 1.12816 330.9632 0.00111 14.23078277 -3.536599e+14 2.121602e+07
176.79548995 243.8675 98.7850 1.12816 330.4900 0.00113 14.23072515 -3.536590e+14 2.121605e+07
Programm trs.c.

> restart; > with(plots): > with(plottools):

Warning, the name changecoords has been redefined
Warning, the assigned name arrow now has a global binding

Constants >
>   R_Eth:=6378000;   # Radius of the Earth
> M_Eth:=5.98E24; # Mass of the Earth
> G:=6.672E-11; # Gravitational const
> mu:=M_Eth*G; #

$R_E th := 6378000$

$M_E th := 0.5981 0^{25}$

$G := 0.6672 10^{-10}$

$mu := 0.3989856 10^{15}$

Input data > fd:=fopen("d:\\sattelites \\date\\alnoaa17.txt",READ);

$fd := 0$

> sNp:=fscanf(fd,"> Np:=op(1,sNp);

sNp := [300]
Np := 300

> for i from 1 to Np do > gr[i]:=fscanf(fd,"> end do: > fclose(fd);

> op(1,gr[1]);

175.8111557

Parameters of linear regression Date (Epoch) -dt Long. ascend. node - anl Argument peregee - arp Mean anomaly - aan Mean value has prefix - s: sdt,sarnl,sarp,saan Mean multiplication prefix - c: ctanl,ctarp,ctaan Mean square - d: ddt

```
> N:=Np; # End index
> Nb:=20; # Start index

        N := 300
        Nb := 20
> sdt:=0;
> sanl:=0;
> sarp:=0;
> saan:=0;
> sincl:=0;
> sp:=0;  samove:=0;  ddt:=0;  ctaan:=0;  ctarp:=0;  ctanl:=0;

        sdt := 0
        sanl := 0
        sarp := 0
        saan := 0
        sincl := 0
        sp := 0
        samove := 0
        ddt := 0
        ctaan := 0
        ctarp := 0
        ctanl := 0

  > for i from Nb to N do
> dt:=op(1,gr[i]);
# Epoch
 >anl:=op(2,gr[i]);
# Longitude ascend. Node
> ecc:=op(7,gr[i]);
# Eccentricity
> arp:=op(5,gr[i]);
# Arg. peregee
> aan:=op(6,gr[i]);
# Mean anomaly
> amove:=op(8,gr[i]);
# Mean motion
> incl:=op(3,gr[i]);
# Inclination
> p:=op(4,gr[i]);
# Focal parameter
> sdt:=sdt+dt;
> sp:=sp+p;
> sincl:=sincl+incl;
> samove:=samove+amove;
> sanl:=sanl+anl;
> sarp:=sarp+arp;
> saan:=saan+aan;
```

```
> ddt:=ddt+dt*dt;
> ctanl:=ctanl+dt*anl;
> ctarp:=ctarp+dt*arp;
> ctaan:=ctaan+dt*aan;
> end do:
> sdt:=sdt/(N-Nb+1);
> sanl:=sanl/(N-Nb+1);
> sarp:=sarp/(N-Nb+1);
> saan:=saan/(N-Nb+1);
> samove:=samove/(N-Nb+1);
> sincl:=sincl/(N-Nb+1);
> sp:=sp/(N-Nb+1);
> f:=sp^2;
#Calculation of linear regression parameters >
>RANL:=evalf((ctanl/(N-Nb+1)-sanl*sdt)/(ddt/(N-Nb+1)-sdt*sdt)); >
RARP:=evalf((ctarp/(N-Nb+1)-sarp*sdt)/(ddt/(N-Nb+1)-sdt*sdt));                    >
RAAN:=evalf((ctaan/(N-Nb+1)-saan*sdt)/(ddt/(N-Nb+1)-sdt*sdt));  >  BANL:=sanl-RANL*sdt;  >
BARP:=sarp-RARP*sdt; > BAAN:=saan-RAAN*sdt;
```

sdt := 252.2973278
sanl := 319.1639384
sarp := -252.9898302
saan := 253.1034345
samove := 14.23140240
sincl := 98.77901032
sp := 1.128127356
f := 1.272671331
RANL := 0.9970098010
RARP := -2.866122188
RAAN := 2.866453539
BANL := 67.6210298
BARP := 470.1251390
BAAN := -470.0951337

Calculation J_2 from long. asc. Node>
> J2anl:=evalf((2/3)*f*(RANL/cos(sincl))/(360*samove));

J2anl := -0.0009164191231

Argument peregee>
> J2anl:=evalf((4/3)*f*(RARP/(1-5*(cos(sincl))^2))/(360*samove));

J2anl := -0.001133217422

Mean anomaly >
> J2ann:=-evalf((4/3)*f*(RAAN/(1-3*(cos(sincl))^2))/(360*samove));
J2ann := -0.001051831107

Create graphics >

Regression line   >

```
>lanl:=line([op(1,gr[Nb]),RANL*op(1,gr[Nb])+BANL],
[op(1,gr[N]),RANL*op(1,gr[N])+BANL], color=red,linestyle=3,thickness=5):
      >larp:=line([op(1,gr[Nb]),RARP*op(1,gr[Nb])+BARP],
[op(1,gr[N]),RARP*op(1,gr[N])+BARP], color=red,linestyle=3,thickness=5):
      >laan:=line([op(1,gr[Nb]),RAAN*op(1,gr[Nb])+BAAN],
[op(1,gr[N]),RAAN*op(1,gr[N])+BAAN], color=red,linestyle=3,thickness=5):
      >pic12:=[seq([op(1,gr[i]),op(2,gr[i])],i=Nb..N)]:
      > pic13:=[seq([op(1,gr[i]),op(5,gr[i])],i=Nb..N)]:
      > pic16:=[seq([op(1,gr[i]),op(6,gr[i])],i=Nb..N)]:
      > pic19:=[seq([op(1,gr[i]),op(9,gr[i])],i=Nb..N)]:
      > pic14:=[seq([op(1,gr[i]),op(4,gr[i])],i=Nb..N)]:
      > pic110:=[seq([op(1,gr[i]),op(10,gr[i])],i=Nb..N)]:
      > pictanl:=[seq([op(1,gr[i]),op(2,gr[i])-RANL*op(1,gr[i])-BANL],i=Nb..N)]:
      > pictarp:=[seq([op(1,gr[i]),op(5,gr[i])-RARP*op(1,gr[i])-BARP],i=Nb..N)]:
      > pictaan:=[seq([op(1,gr[i]),op(6,gr[i])-RAAN*op(1,gr[i])-BAAN],i=Nb..N)]:
      >ptanl:=pointplot(pictanl,labels=[day,degree],font=[SYMBOL,2], title="Long variations of
longitude ascending node "):
      >ptarp:=pointplot(pictarp,labels=[day,degree],font=[SYMBOL,2], title=" Long variations of
argument peregee "):
      >ptaan:=pointplot(pictaan,labels=[day,degree],font=[SYMBOL,2], title=" Long variations of
mean anomaly "):
      > p1:=pointplot(pic12,labels=[day,degree],font=[SYMBOL,2], title="Secular variations of
longitude ascending node "):
      > p3:=pointplot(pic13,labels=[day,degree],font=[SYMBOL,2], title=" Secular variations
argumenr peregee"):
      > p6:=pointplot(pic16,labels=[day,degree],font=[SYMBOL,2], title=" Secular variations
mean anomaly"):
      > p4:=pointplot(pic14,labels=[day,"p/R_Eth"],font=[SYMBOL,2], title=" Secular variations
of focal parameter "):
      > p9:=pointplot(pic19,labels=[day,"Dj"],font=[SYMBOL,2], title="Variations of specific
energy "):
      >p10:=pointplot(pic110,labels=[day,"m^2/s"],font=[SYMBOL,2],    title="Variations    of
specific angle moment "): > plots[display](ptanl);
      > plots[display](ptarp); > plots[display](ptaan); > plots[display](p1,lanl);
      > plots[display](p3,larp); > plots[display](p6,laan); > plots[display](p4); > plots[display](p9);
> plots[display](p10);
```

Chapter 2.   Sun

2.1 Theoretical background

2.1.1 Sunspots and their motion on the solar disk

(The theory is given in accordance to the course Life of the Earth in the solar atmosphere. For detailed information one should appeal to this course and to literature given in the section List of literature)

Dark sharply outlined sunspots forming groups are the most noticeable structures in visible light (Fig. II-1). Among the number of more or less small spots two big spots are usually distinguished. They form a bipolar group with the opposite polarity of the magnetic field and with the intensity equal to thousands of oersted .
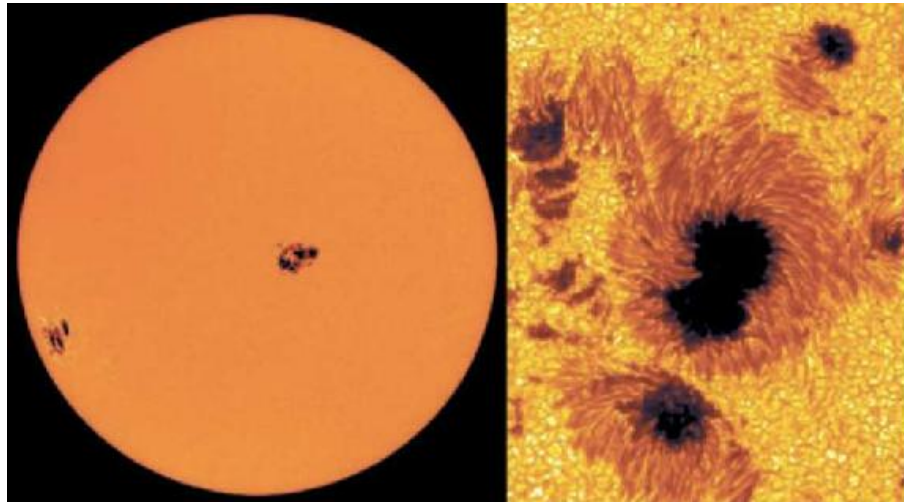

Fig. II-1. Groups of Sunspots

Sunspots appear in a form of dark pores between granules (see inscription and arrow in fig. 2) having size of several thousands of kilometers. The shadow of a developed big spot has a diameter of tens of thousands kilometers and brightness 5-10 times less than the surrounding photosphere. The spot shadow is surrounded by a radial structure.

Sunspots are the places where a strong magnetic field of $(2{-}4) \cdot 10^3$ oersted coming out into the photosphere. The shadow temperature is $(3{-}4) \cdot 10^3$ K. for the first time a vertical motion of plasma with the speed of 6 km/s in semishadow was discovered by English astronomer Evershed in1909. The dimensions in million kilometers (megameters, Mm) are laid off along axes
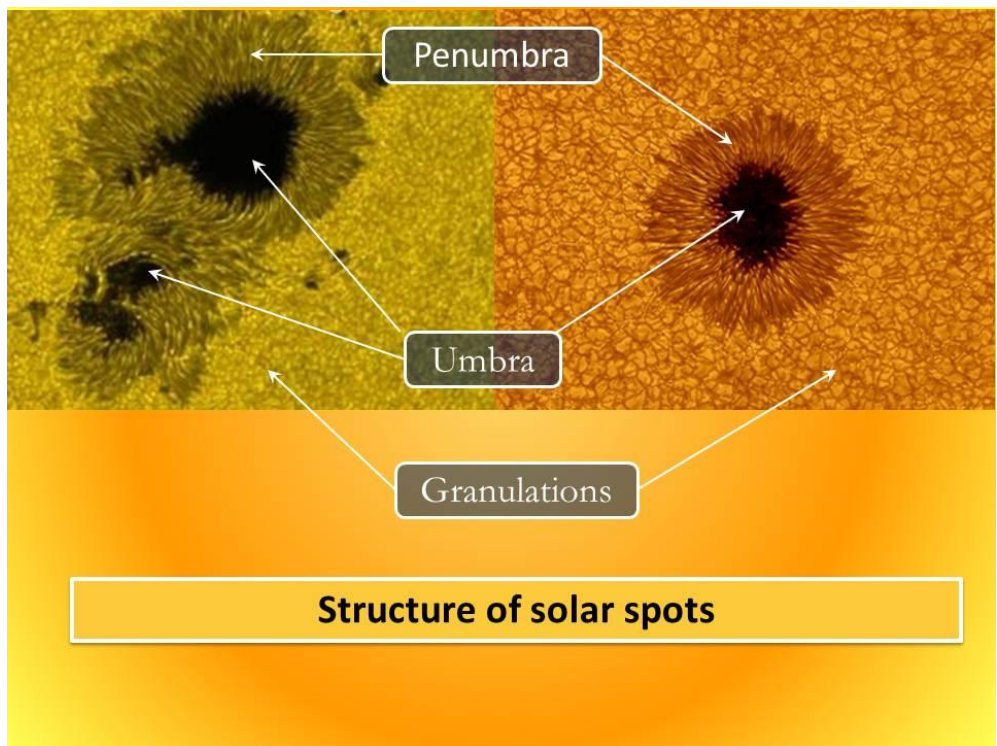
Fig. II-2. Basic elements of Sunspots structure.

Spots have quite stable dynamics and cinematics.

First, spots at different latitudes take part in general rotation of the Sun which differs from the angular velocity at different latitudes. This phenomenon is called the differential rotation of the Sun. A visible motion on the solar limb is related to this phenomenon.

Second, spots appear as small dark areas on the Sun grow under a certain law and then reduce and disappear.

The lifetime of big size spots is long enough so that within this period a spot has time to complete the turn around the Sun which takes 27 days.

The physics of spot formation and disappearance processes is not studied yet. So it is useful to examine some simple laws of the process to have a general idea of the character of this process.

Model of Sun spots evolution

In this task laws of time variation of spot area and other parameters are explored. The task is targeted at the verification of a standard model of sunspot area variation based on the data obtained from the examination of sets of photo images of the Sun.

The standard model represents a so-called log-normal distribution:

$$S(t) = S_0 \exp\left\{-\frac{(\ln(t/t_0))^2}{2\sigma^2}\right\}.$$

Here $S(t)$ is spot area as a time t function, t_0 a moment of time when the area is maximum, σ^2 a quantity characterizing the lifetime of a spot, S_0 - maximum area of a spot. With that it is considered that the moment $t = 0$ corresponds to the birth moment of a spot. Typical graphs of this dependence are shown in fig. II-3
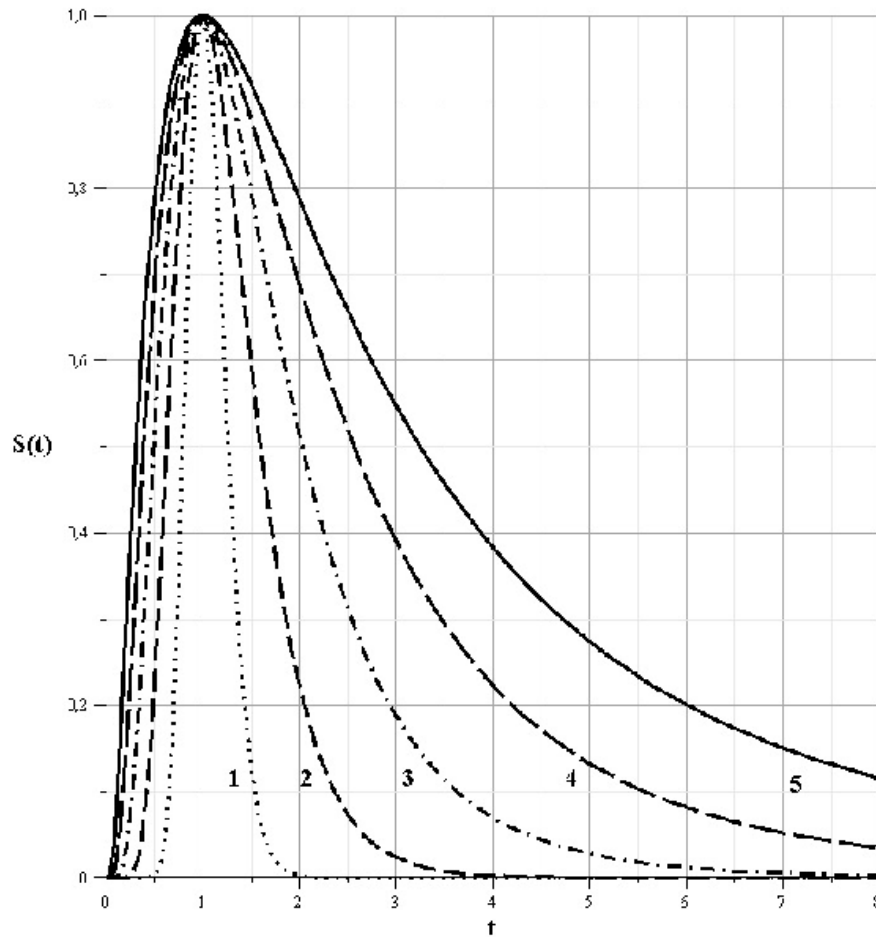
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Fig. II-3. Log-normal distribution for different values of dispersion σ^2: 1 - $\sigma = 0.2$, 2 - $\sigma = 0.4$, 3 - $\sigma = 0.6$, 4 - $\sigma = 0.8$, 5 - $\sigma = 1.0$.

In fig. II-3 curves of log-normal distribution are shown for five values of parameter σ for values: $S_0 = 1, t_0 = 1$. As it is seen from graphs log-normal description describes the process of fast birth of spots and slower destruction.

Another model is a model (Lorentz distribution):

$$S(t) = \frac{S_0}{\sigma^2 + (t/t_0)^2}.$$

Here parameters t_0, σ have the same sense as for log-normal distribution. Maximum area of a spot is $S0/\sigma^2$. This distribution is represented in fig. II-3.
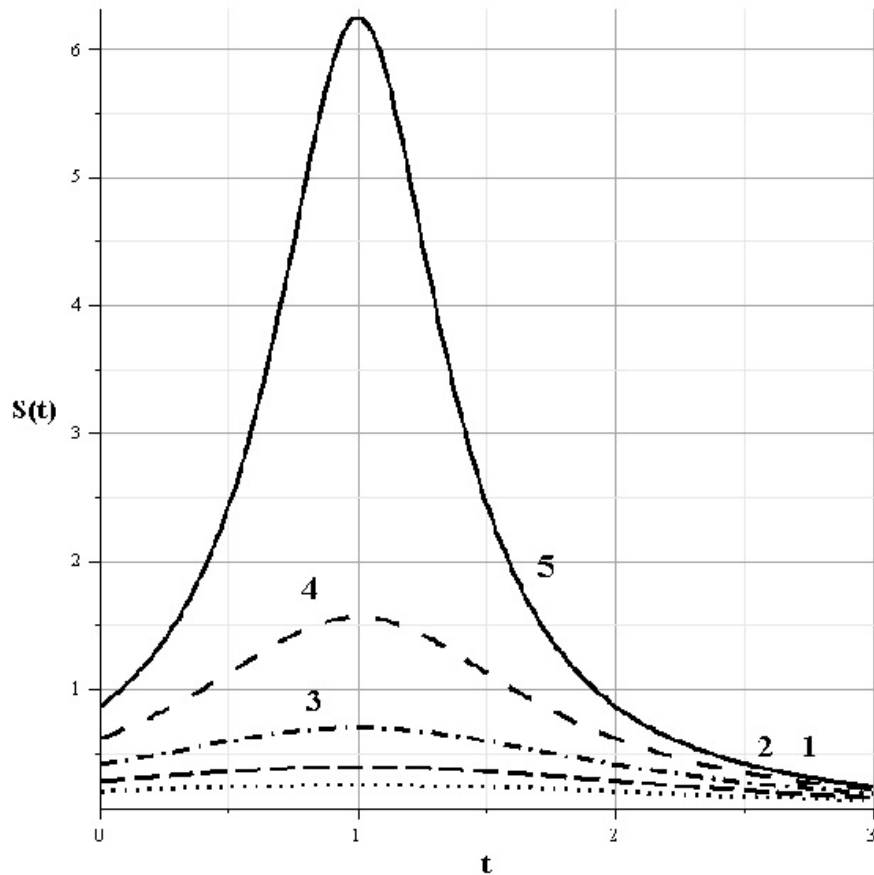
Fig. II-4. Lorentz distribution for different values of parameter σ^2: 1 - $\sigma = 0.4$, 2 - $\sigma = 0.8$, 3 - $\sigma = 1.2$, 4 - $\sigma = 1.6$, 5 - $\sigma = 2.0$.

It is clear from the comparison of figures II-3 and II-4 that Lorentz distribution describes the process of appearance of sunspot with the same speed as its further destruction. In the offered problem the best of the two models is to be chosen by the Fisher criterion.

The procedure of spots parameters calculation

.

The procedure of identification of spots on the solar limb is based on a special procedure of contouring areas bounded by a curve with given value of color level, defined by RGB-representation of a color dot of the image. The program identifies such areas automatically and calculates their area as a number of pixels of the image.

Area measured in pixels is converted into metric area with the help of a simple correspondence between linear dimensions of the Sun in the picture and real Sun dimensions. To do this the program does the calculation of the solar diameter in pixels as the diameter of a circle. The circle is plotted basing on the boundary points of the solar limb in the picture with the help of the least-square method. The relation of the real solar diameter in km to the diameter in pixels yields a parameter for conversion of pixel area into real area. More over the program makes the correction of the spot area by the angle of view. As far as the majority of spots have relatively small angular dimensions, the correction of the spot area can be done by the following equation with high accuracy :

$$S = \frac{S'}{\cos \lambda \cos \theta},$$

where S in corrected area, S' - not corrected area, θ - the latitude of the spot center counted from helioequator (the middle line of the image), λ longitude of the spot center counted from the central meridian of the image (the center of a picture). With that the center of an image is detected as the center of the smallest rectangle, completely bounding the spot.

Coordinates of the center of a spot are traced in all totality of images to be processed. Motion of a spot is detected by these coordinates.

The analysis of the longitudinal motion yields the estimation of the speed of the differential rotation at the given latitude. Latitudinal motion of a spot gives the estimation of a current inclination of the solar axis to the view axis. Maximum of this inclination is about 70 degrees.

2.1.2 The differential rotation of the Sun

The differential rotation means that the Sun rotates not as a solid body but as a gas sphere with the different speed of contrasting elements on its surface (for example spots) at different distances from the solar equator:

$$V(\theta) = \Omega_0 (1 - b\cos^2(\theta) - c\cos^4(\theta)).$$

At the equator the rotation velocity is about 2 km/s and decreases to the Poles. The general form of this dependence is usually given as: $\Omega_0 = 2.87 \cdot 10^6 \ \mathrm{rad \cdot s^{-1}}$, $b = 0.12$, $c = 0.17$.

The problem of the determination of velocities of the differential rotation of the Sun at different helio-latitudes is usually solved with the help of long-period observation of separate spots within a week or even longer. The period of time needed for a spot to complete the total turn at this latitude is counted. After that the velocity is calculated as a ratio of the length of a travel arch of a spot to the time of travel. The velocity value is usually considered as the average one for a set of spots. Such procedure requires an analysis of a great number of Sun images obtained in quite a long period of time.

In the given task another method is offered. It allows us to estimate the speed of the differential rotation of the Sun in quite a short period of time of several hours. The basis of the method is in the measurement of phase velocity of harmonic components visual band electromagnetic radiation intensity variations or variation of the magnitude of the solar photosphere magnetic field in zonal direction (along the equator). If we observe quite stable-shape spots for several hours, their harmonic components will have phase velocity coinciding with the velocity of the whole spot. More over a compact shape of a spot in zonal direction in the intensity of radiation leads to the fact that there will be several separate harmonic waves with different zonal wave numbers in Fourier expansion in harmonic components and therefore these components will have different frequencies. With that phase velocities of those components will be equal.

With that the spectrum will also contain harmonic components with phase velocity different from the velocity of the whole spot. Such waves are related to different wave processes possible in the solar atmosphere. For example harmonic components can be related to the generation of hydrodynamic waves in plasma at boundaries of flows moving with different velocities. The example of such wave generation is shown by waves in atmospheres of the giant planets. Such waves also exist in the terrestrial atmosphere and oceans.

The target of this work is the calculation of the phase velocity of sets of harmonic components of radiation intensity variation in different bands of electromagnetic radiation and magnetograms of the Sun with the help of estimation of these variations time-space spectra.

Wave numbers and frequencies spectra

To calculate wave numbers and frequencies of harmonic components of radiation intensity or magnitude of the magnetic field (further intensity) an antenna array method is usually applied. An antenna array is usually a set of elements (nodes of antenna array) placed in the space at some well-known distances. There is a receiver or detector in each node. It measures the magnitude of a physical parameter in time-space variation of which waves are observed. This can be the intensity of electromagnetic radiation (as it is in radiotelescopes), the pressure of the atmosphere or magnitude of the magnetic field. Time series of separate magnitudes of a parameter measured at the same moments of time in all nodes are usually used for the exploration of frequencies. Spectral matrixes are calculated in the frequency range from 0 to the Niquist maximum frequency determined by the formula $f_n = 1/(2\Delta t)$ where Δt is a time interval between consequent measurements of the parameter. Except series autospectra the spectral matrix contains cross-spectra which allow the estimation of phase shift between harmonic components with one and the same frequency at different elements of antenna array. The magnitude of these phase shifts is related to the value of wave number of a harmonic component in given frequency. Details can be found in [13, 13, 15].

It is convenient to change this procedure in this task and change places of time and space. As time series it is possible to use spatial series of intensity in zonal direction taken from separate images of the Sun. Separate images (photos) can be considered as a set of variations at the nodes of antenna array which is placed in the time. Nodes of antenna array are the moments of time when corresponding images were obtained. It is supposed that the time of image reception is considerably less than the interval between images. Such replacement of the time and the space becomes necessary because the intervals between separate images are not usually regular and the number of images does not exceed ten per day. That is why it is impossible to get a set of points big enough for a spectral estimation. On the contrary the series of points in zonal direction are regular and unchangeable for separate images. A standard resolution for images 512x512 or 1024x1024 pixels allows to obtain a set of data representative enough for the estimation of peak wave numbers in the intensity spectrum along helio-longitude for each image.

The calculation of wave numbers and frequencies of harmonic components of radiation intensity contrasts or a magnitude of the magnetic field are usually done in two steps according to common principles of spectral analysis. At the first step a spectral matrix is built $S(\theta, k) = (S_{ij}(\theta, k)), i, j = 1, \ldots, M$ as a function of zonal wave numbers k for each fixed latitude θ in the totality of images of the Sun. here M is a number of antenna array elements and in this case it is a number of separate processed images. For each latitude an antenna array is a sequence of time moments when separate images were obtained.

That is why the spectral matrix is built on the totality zonal series taken for a fixed latitude of each chosen image of the Sun. To make an assessment of spectral matrix $S(\theta, k)$ in this work a method of maximum entropy is used. It is related to multidimensional autoregression (see details in the part Methods of spectral estimation). A typical form of wave numbers spectra is represented in the following picture. After the calculation of a spectral matrix all local maxima of autospecra $S_{jj}(k), j = 1, \ldots, M$ are found (diagonal elements of the spectral matrix). Among all local maxima those are distinguished which have a mean coherence higher than the critical value $\chi_{cr}(k) = 0.8$. for a given wave number k coherences χ_{ij} and phase shifts ϕ_{ij} between nodes of antenna array are related to nondiagonal elements of the spectral matrix by the following relation: values of coherences $\chi_{ij}(k)$ lie in the interval [0,1] and describe the degree of linear connection of harmonic components between elements of antenna array i and j for a given wave number k. the error in the definition

of phase shift considerably depends on the coherence magnitude. So for the purpose of this problem it is useful to consider only the spectrum characteristics near the high coherent harmonic components.

At the second step spectrum of frequencies S $S(\theta, k, f)$ is built with the help of spectral matrix. In this work the creation of a frequency spectrum is realized either with the help of maximum-likelihood method or Bartalett method with a glance at interferential effects or without it. On default a normalized spectral matrix with elements is used to create a spectrum. In this case interferential effects are excluded. This method of estimation should be considered as the best (see. Methods of spectral estimation). A typical set frequency spectra is represented in the following figure. Having estimated S $S(\theta, k_a, f)$ for each $k_a, a = 1, \ldots, n$ from a list of wave numbers (n - is a number of peaks in the spectrum) corresponding to high coherent peaks in a wave number spectrum, one has to find the absolute maximum of spectral density $S(\theta, k, f)$ with respect to frequency. As the result a list of pairs of wave numbers and spectral density peaks frequencies: the union of all these point at a graph ((k, f) allows the creation of a dispersion curve which should be approximated by a linear function. The inclination of a linear regression function equals to the needed phase velocity. This velocity in turn defines the rotation velocity at this latitude. If there are only harmonic components in the spectrum which are related to the intensity contrasts motion as the whole in zonal direction, then this velocity coincides with the velocity of differential rotation.

The diagram wave number - frequency built on seven photos for two latitudes 46.4 and 40.1 degrees of northern latitude. The red lines show lines of linear regression of peaks in frequency spectrum for a list of wave numbers at each latitude.

Obtaining the dots intensity of the image

In this problem a simple method of obtaining the intensity is accepted. It is based on summing the intensities of red, blue and green colors in RGB-representation of dots. This method is justified by the fact that the color of a dot does not play any important role in this task, but a motion of dots with the same values of color is interesting. It is useful to exclude one or two color layers from RGB-representation to improve the contrast. So the program offers a user a possibility to choose the best combination of layers with the help of switching on and off any layer by a switch placed in the first inlay Solar. Three switches have a shared name RGB-filter and are marked with respective colored labels. It is important that at least one layer is switched on to create a set of data with nonzero colors. It is recommended that all three layers are switched on for grey images. It is recommended for example for solar magnetograms.

Algorithm of distinguishing the solar dimensions in the photo

Algorithm of distinguishing the solar dimensions in the photo is different for different types of solar photos

For MDI images (visible band) and MDI-mag (magnetograms) the algorithm represents the finding of a dot in the chosen line (line 256) and the total brightness of RGB-intensity of this dot must exceed the background intensity chosen to be equal to 10. The extreme dots with such exceeding on the left and on the right are the extreme points of a solar disk. The real diameter of the Sun is 1320000 km. the relation between the real diameter in kilometers and the pixel dimensions yield the relation between all the major metric scales of the solar image in the picture and its real dimensions. The vertical diameter of the Sun is defined in the same way.

For ETI-images the algorithm is different. It is related to the fact that the solar corona is visible outside the disk in these photos. The criterion of the edge of solar disk is local maximum of

RGB-intensity magnitude on the right and on the left. As far as local extremums may appear due to local variation of intensity, it is necessary firstly to smooth the dots intensity in a line to determine the edge. So at the first step a smoothing of intensities is realized with the help of cosine filter in a chosen line (line 256) and then left and right positions of the disk edges are determined independently with the help of a simple algorithm of local extremum distinguishing. As in case of MDI-images the solar disk dimensions are chosen to be equal to the pixel distance between the left and the right maxima. The vertical solar diameter is defined in the photo in the same way .

Calculation of the wave phase velocity.

The calculation of the phase velocity is based on a standard determination of a wave phase velocity as a ratio of a wave number to its frequency. The estimation of the wave number is realized in the program with the help of wave numbers spectrum by the positions of spectral density peaks in zonal wave numbers scale. At the next step for each peak a frequency spectrum is estimated. A frequency corresponding to the peak position in autospectrum is considered as a wave frequency. The relation of these quantities is a normalized phase velocity. As the basic verifiable model of the differential rotation of the Sun such model is used: constants A and B are to be defined with the help of least squares. But as it is shown by a theoretical analysis the model can be chosen in more generalized form. So a user is able to vary the form of the model but justifying his choice.

2.1.3   The solar wind, SCR and Parker's spiral

The area around the Sun with the radius about 75-90 a.u. is called heliosphere. There solar magnetic field and flows of charged particles from the Sun prevail over the field and particle of galactic origin. Heliopause is an area where ions of solar and galactic origin collide. This area has a radius of about 110 a.u. The area placed just behind the heliopause represents a shock wave in the galactic gas in front of the moving Sun. This area stretches up to the radius of about 230 a.u. Schematically these areas are represented in fig. 1.

All basic processes generated thanks to the solar activity and influencing the Earth are concentrated in the area of heliosphere placed between the Sun and the Earth. Solar cosmic rays (SCR) and the solar wind play the most important role in solar-terrestrial relations. SCR and solar wind represent charged particles of different energy.

One more important element of the heliosphere is the magnetic field generated by the Sun. this problem is devoted to the exploration of these objects and their interconnection.

The solar-terrestrial physics considers the relation of phenomena taking place on the Earth and in the near-Earth space to the solar activity.

The Sun rotates around the axis deviating from the normal to the plane of ecliptic by $7^o 15'$ and with that the angular velocity measured in degrees per day decreases with the growth of heliographic latitude.

The corresponding period (siderical period) is about 25 days at the equator and reaches 30 days near the poles. The Earth moves around the Sun in the same direction and the period of rotation of the Sun relative to the observer on the Earth (synodic period) is almost 27 days and 32 days near the poles. At the equator the linear velocity of rotation is about 2 km/s.

The energy flux from the Sun heats the coronal plasma to 100 eV and is not equalized by the coronal radiation (due to its low density), i.e. the solar corona is not in the state of hydrodynamic equilibrium and constantly expands. The extension of the solar corona into interplanetary space (up to 100 a.e.) is called heliosphere and the radial flows of plasma taking away the excess of energy is called the solar wind.
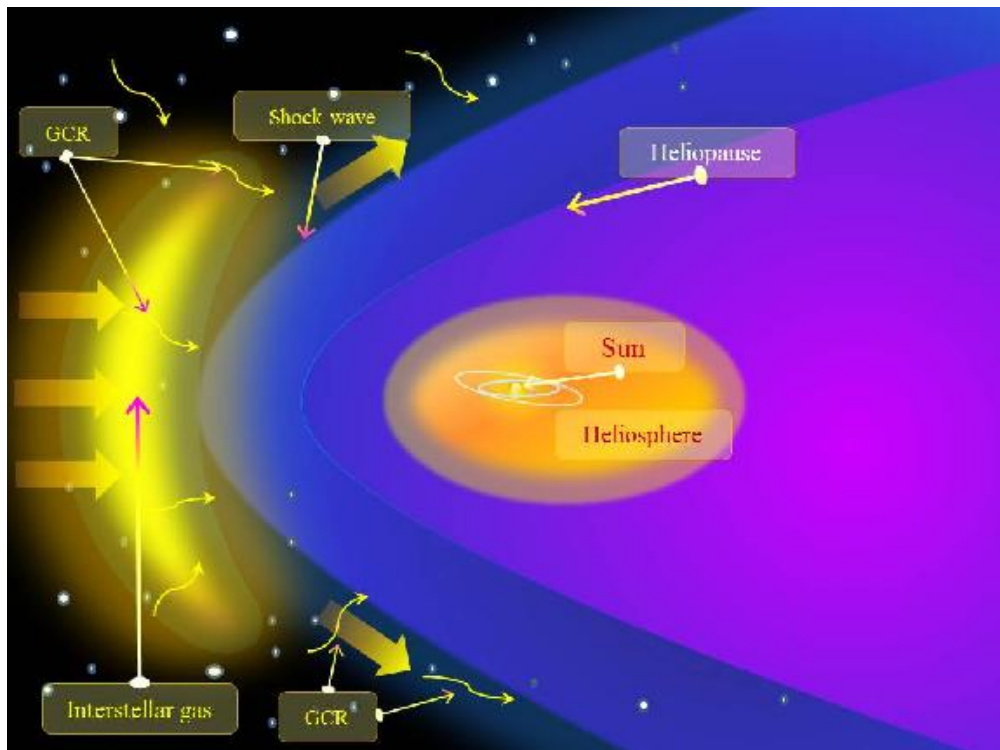
Fig. II-5. The structure of the vicinity of the Solar system interacting with the galactic medium.

The motion of charged particles in heliosphere (SCR) is performed along the Parker's spiral representing the spiral of Archimedes. The general conception of such spiral can be seen in fig. II-6.
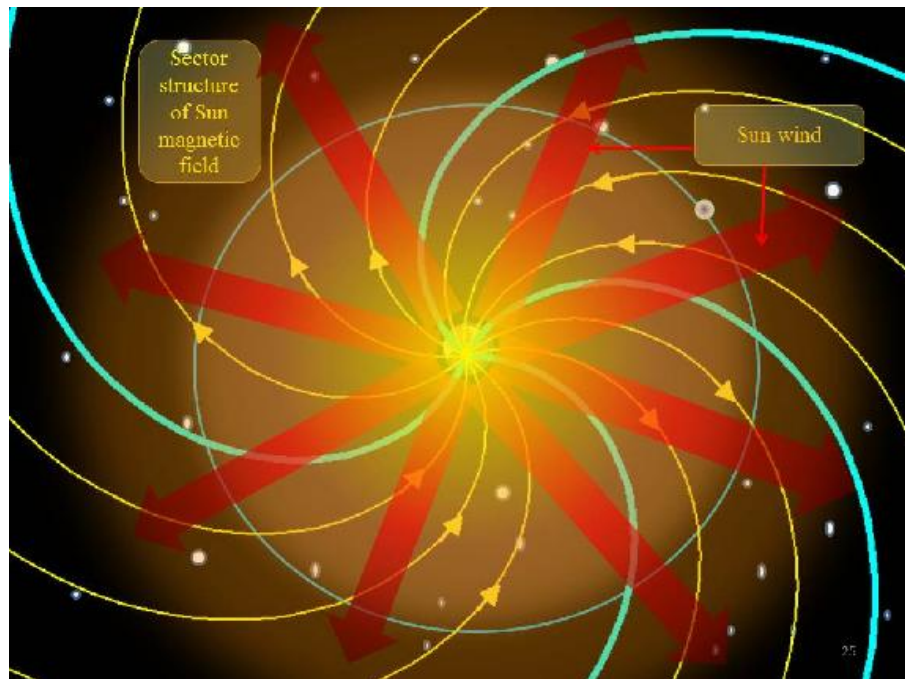

Fig. II-7 the sectoral structure of the solar magnetic field.

It is shown in fig. II-4 that the solar wind moves radially. The solar wind stretches lines of force in radial direction. Lines of force rotate together with the Sun and it males the spiral of Archimedes which is called Parker's spiral in this case. The spiral is described by the equation:

$$r = \frac{u}{\Omega}(\varphi - \varphi_0) + r_0.$$

Here u in the speed of solar wind, Ω is angular velocity of solar rotation at the equator, φ is angle in the coordinate system with the center on the Sun corresponding to the observer's position, r is radial distance from the observation position to the center of the Sun. and φ_0 is the initial angle of the spiral branch on the surface placed at the distance r_0 from the center of the Sun.

During flares X-ray radiation gets to the Earth in 8 minutes if it propagates directly. It is possible to compute the point on the Sun where the flare happened if we measure the time of arrival of SCR particles from the flare registered by an X-ray burst. For this a length of the Archimedes' spiral the particles move along is needed. It is described by the formula:

$$L = \int_0^\varphi k\sqrt{1+(\varphi)^2}\,d\varphi = \frac{k}{2}\left[\varphi\sqrt{1+\varphi^2} + \ln(\varphi+\sqrt{1+\varphi^2})\right] \tag{2.1}$$

In this formula:

$$k = \frac{u}{\Omega}.$$

This is the distance between the center of the Sun and the point of arrival of SCR particle. It is necessary to know the speed of the solar wind at the given moment of time to calculate the point where the burst has come from. If one chooses particles with given energy as a time marker of SCR arrival and measures the time of their travel to the Earth it is possible to compute the distance they have travelled. If we know this distance it is possible to calculate the angle of turning of the spiral (using (2.1)) and find the point on the Sun.

2.2   Tasks of practical works

2.2.1   Task 7. Dynamics and cinematics of sunspots

A problem of computing of cinematic motion along the solar disk is a simple but important exploration of sunspots. Mostly this motion is related to the differential rotation of the Sun. this rotation was discovered by the examination of dependence of the motion velocity on the heliolatitude. It is offered to calculate the motion velocity of chosen spots on the Sun and estimate the period of completion of the whole turn by the spot independently in this problem. A set of images of a several days period is need, then it is necessary to estimate the position of the conventional center of a spot and compute its motion in heliolatitude and heliolongitude. The motion in the heliolongitude is mostly related to the differential rotation and the motion in the heliolatitude is related to the inclination of the solar rotation axis to the plane of ecliptic. It is possible to estimate the projection of the solar rotation axis on the plane of visible disk of the Sun by the maximum displacement of the spot along the heliolatitude and then calculate the angular distance of the Earth from the current position to the positions on the orbit where the visible inclination of the solar rotation axis will be zero and maximum. To complete this task it is possible to perform the calculations independently or use the program package ``Cosmophysics'' (the problem ``Dynamics and cinematics of spots'').
<u>Task.</u>
1. From the database of images of the Sun obtained with the help of spacecraft SOHO, SDO or Hinode choose a sequence of images in a period of 2-3 months. The choice should be done in such way that the number of spots should not be too big and spots should be placed at different

heliolatitudes.

2. Calculate positions of spots in several days and graph the dependence of heliolatitude and heliolongitude on time.

3. Calculate the motion velocity of spots in the heliolongitude during the examined period. Estimate the time needed for a spot to make a complete turn and come out at the eastern edge again. Investigating the sequent images of the Sun define if the spot appeared in supposed time. Explain the result.

4. Calculate the maximum displacement of spots in heliolatitude within the given period. Estimate a visible inclination of the solar rotation axis for spots at different heliolatitudes. Compute the angular distance of the Earth on the orbit where the visible inclination is zero or maximum. Estimate the time neede for the Earth to travel this distance. Verify the result with the help of images obtained in the calculated time. Calculate the error of this estimation.

2.2.2 Task 8. Dynamics of the sunspots area

One more important element of spots exploration is the examination of time variation of their area. This information is important because there is no exact information about the formation of spots and their deep structure. A simple research able to give some useful information concerning the process of sunspots formation is an examination of time variation of their area. Results obtained in problem 7 can point at the importance of such work. This problem lets answer the question - why the spot appeared or did not appear on the solar disk after the complete turn around the Sun. if the spot appeared it is possible to answer another question - is it a new spot or an old one. The answer to this question is unequivocal. But the forecast of the spot area offered to be done in this problem can serve as the information for this answer. To complete this task it is possible to perform the calculations independently or use the program package ``Cosmophysics'' (the problem ``Dynamics and cinematics of spots'').

Task.

1. From the database of images of the Sun obtained with the help of spacecraft SOHO, SDO or Hinode choose a sequence of images in a period of 2-3 months. The choice should be done in such way that the number of spots should not be too big and spots should be placed at different heliolatitudes.

2. Calculate the positions of spots in first several days and their area. graph the time dependence of the spot area.

3. Use one of the models of spot area variation and estimate parameters of these models with the help of least-squares method. Explain the result.

4. Using the obtained parameters of the model estimate the spot area by the moment of its appearance at the eastern edge of the solar disk . compare it to real images. Explain the obtained result.

2.2.3 Task 9. The differential rotation of the Sun

Method of completion the tasks

Method of completion the tasks consists of three stages. The first stage is selection of photographs needed for the determination of the differential rotation. The second stage is the initial processing in order to get the information about dispersion relation of zonal waves. The third stage is modeling of the differential rotation. The first two stages should be completed with the help of ``Cosmophysics'' program and the last stage should be completed independently with the help of any mathematical calculation system of with the help of a written program.

The sequence of operations can be described as the following.

1. Input of the list of solar images and the indication of the region parameters given by a teacher. This procedure is described in chapter ``Initial information''.

2. Formation of the initial set of data. This procedure is described in the chapter ``Input set of data''. Its preparation consists of two stages: ``Specification of RGB-filter'' and ``Filtration of series''.

3. For each latitude from the data area there is step-by-step procedure of calculation of maximums of wave time-space spectrum, which propagate in zonal direction. The procedure includes several stages. For each latitude:

3.1. The first stage is the preparatory selection of the optimal degree of the model.

3.2. Calculation of a spectral matrix as a function of a wave number and definition of positions of spectral density maximums in the examined interval of wave numbers with an average coherence more than 0.8.

3.3. For each selected maximum of wave numbers spectrum an estimation of frequency spectrum and definition of frequency spectral maximum positions should be done.

3.4. Memorizing of the output set of data.

4. Estimation of dispersion curves inclination with the help of least-squares method and calculation of differential rotation velocity with the help of the output set of data. Calculation of the model curve of the differential rotation and estimation of errors of model parameters definition.

5. Representation of results in a form of report and graphs.

Initial information

Sets of solar images in MDI-band or megnetograms are used as the initial information for the solution of the problem. The choice of list of images and their type is defined by a teacher before the work when the permission for the realization is obtained. It is recommended to use magnetograms because they have the most contrast elements of structure. All images should be converted in BMP format with a size 512x512 pixels. That can be done with the help of any graphic editor.

Input of images should be realized in a sequence or as a selected list when the button Add is pushed. The button is placed in the inlay Solar just below the window List of Solar Images. After this button is clicked a standard dialog of image input opens. A needed catalogue should be selected and uploaded into the program memory by a click on a needed file. After the dialog is opened images can be uploaded in a list which can be selected in a dialog with the help of Shift button held during the motion of cursor by ``arrows'' buttons. In the group of control elements ``Images of the Sun'' there are also three more buttons except the button Add: Delete, Order, Clear. Button Delete is needed for the removal of a single file from the list; button Order sorts the list of images by the time or creation of an image. It is useful for a convenient viewing of images on a display. The button Clear is needed to clear the list of images before downloading another list.

The totality of images should be placed in a separate catalog. After its download with the help of elements group List of images of the Sun placed in the first inlay Solar, the list of images can be memorized on a disk and can be downloaded during the second use of program. But it is recommended only after the selection of the processing region of the image totality because the list of images (*.LST format) also contains the information about the region of data by the moment of saving. With that one should remember that the list of images contains files names and routes to the catalog where images are saved. So the change of route to the catalog will lead to the failure of file search on a disk. This fault can be debugged by the correction of list file (see ``file structure List of images'' *.LST). saving the list of images is realized by a click on a display button [PICTURE SAVE.BMP] on the control panel (or in a file menu File). Repeated download can be realized with the help of display button [PICTURE OPEN.BMP] on the control panel (or in a file menu File). File

structure ``List of images''.

Information about the list of images is saved on a disk in a special text file with LST extension. The example of this file is:

6

D:\DATA\Magnitometer\2001\BMP\BMP-512\ 31

MDI_mag_2001.01.01_00	03_resize	2001.01.01.00.03.00	63146908980	31
MDI_mag_2001.01.01_01	36_resize	2001.01.01.01.36.00	63146914560	31
MDI_mag_2001.01.01_03	12_resize	2001.01.01.03.12.00	63146920320	31
MDI_mag_2001.01.01_04	48_resize	2001.01.01.04.48.00	63146926080	31
MDI_mag_2001.01.01_06	27_resize	2001.01.01.06.27.00	63146932020	31
MDI_mag_2001.01.01_08	03_resize	2001.01.01.08.03.00	63146937780	31

Line 1. Contains the number of images in a list (in this case it is 6).

Line 2. Contains the route to the catalog where the list of images without file names is saved.

Lines 3-6. Each line is used for its own image and contains sequent information in a form of substrings separated by at least one space:

Substring 1. An integral number denoting the length of file name (in this case 31)

Substring 2. File name (without the route) with the length indicated in the first substring.

Substring 3. Time of creation in a format:

YYYY .MM .DD.OO.MM .SS

(YYYY-year, MM - month, DD - day, OO - hour, MM - minute, SS - second)

Substring 4. The time of creation of the image in time units established by a variable in the last line of the file, counted from the one of standard moment of time, e.g. 0000.00.00.00.00.00. in this case the time is counted in seconds.

Line 7. Contains the beginning and the end of the data region in longitude, counted from the left edge of the image (in this case 156356).

Line 8. Contains the beginning and the end of the data region in latitude, counted from the upper edge of the image. (in this case 136 376).

Line 9. Contains step in longitude for a choice of dots (in this case 10).

Line 10. Contains step in latitude for a choice of image lines for a set of data (in this case 20).

Line 11. Contains time dimensions type (in this case 0).

Output set of data. The example of output file created by the program at the end of work:

D:\DATA\Magnitometer\2001\1-7.lst 7 D:\DATA\Magnitometer\2001\BMP\BMP-512\ 31

MDI_mag_2001.01.01_00	03_resize	2001.01.01.00.03.00	63146908980	31
MDI_mag_2001.01.01_01	36_resize	2001.01.01.01.36.00	63146914560	31
MDI_mag_2001.01.01_03	12_resize	2001.01.01.03.12.00	63146920320	31
MDI_mag_2001.01.01_04	48_resize	2001.01.01.04.48.00	63146926080	31
MDI_mag_2001.01.01_06	27_resize	2001.01.01.06.27.00	63146932020	31
MDI_mag_2001.01.01_08	03_resize	2001.01.01.08.03.00	63146937780	31
MDI_mag_2001.01.01_09	39_resize	2001.01.01.09.39.00	63146943540	

Parameters of data region (pix) (pix) 156 356 136 376 1 20 0

Real Sun dimensions: $1.392 \cdot 10^{+06}$ km

Horizontal Sun dimensions: 497 pix

Vertical Sun dimensions: 497 pix

Latitude: 0 (120 , 28.8748)

The last model rang: 6

Wave number range: [0, 500];

Frequency range: [-0.5 , 0.5];

Frequency step: 0.01

Normalized wave number: 0.043 -0.39 -0.23 -0.03 0.17 0.3 0.45 0.130258 0.137045 0.221468 0.128913 0.130603 0.129564

Normalized wave number: 0.082 -0.45 -0.15 0.04 0.21 0.36 0.132586 0.179725 0.152298 0.13264 0.131843

Normalized wave number: 0.11 -0.36 -0.16 0.02 0.16 0.31 0.45 0.136079 0.215618 0.13309 0.130956 0.13022 0.130672

Normalized wave number: 0.169 -0.26 -0.06 0.1 0.24 0.39 0.205439 0.134387 0.131355 0.130169 0.131585

Normalized wave number: 0.222 -0.36 -0.17 0.1 0.26 0.42 0.188549 0.139529 0.137287 0.130745 0.140161

Normalized wave number: 0.258 -0.47 -0.13 0.02 0.31 0.19449 0.131373 0.133878 0.133967

Line 1. Contains route and the name of images list file.

 Line 2. Contains number of processed images.

Line 3. Contains route to the list of images.

Lines 4 - 10 . (the number of these lines is determined by the parameter in line 2). Contain the following information in each line.

Substring 1. An integral number indicating the length of file name (in this case 31)

Substring 2. File name (without the route) with the length indicated in the first substring.

Substring 3. Time of creation in a format : YYYY.MM.DD.OO.MM.SS

(YYYY - year, MM-month, DD - day, OO - hour, MM - minute, SS second)

Substring 4. The time of creation of the image in time units established by a variable in the last line of the file, counted from the one of standard moment of time, e.g. 0000.00.00.00.00.00. in this case the time is counted in seconds.

String 11. Contains the beginning and the end of the data region in longitude, counted from the left edge of the image (in this case 156356).

Line 13. Contains step in longitude for a choice of dots (in this case 10)

Line 14. Contains step in latitude for a choice of image lines for a set of data (in this case 20)

Line 15. Contains time dimensions type (in this case 0).

Line 16. Contains solar diameter used in calculations.

Line 17. Contains horizontal solar disk dimensions in pixels for a given image totality (in this case 497)

Line 18. Contains vertical solar disk dimensions in pixels for a given image totality (in this case 497) The lines follow which contain the same information for each latitude:

Line 19. Contains number of latitude and latitude in degrees and in pixels in brackets (in this case respectively 0 (120, 28.8748)).

Line 20. Contains the last matrix rang when a spectral matrix was estimated at this latitude: 6

Line 21. Contains wave numbers range where a spectral matrix was estimated: [0, 500]

Line 22. Contains frequency range where frequency spectrum was estimated: [-0.5, 0.5] and frequency step: 0.01.

Line 23. Contains normalized wave number for which a frequency spectrum was estimated: 0.043

Lime 24. Contains the list of spectral density magnitude in frequency spectrum peaks for a given wave number -0.39 -0.23 -0.03 0.17 0.3 0.45

Lime 25. Contains wave number peaks : 0.130258 0.137045 0.221468 0.128913 0.130603 0.129564

Then the lines follow containing the same information for all wave number peaks starting from line 23 and finishing by line 25.the the information repeats for the next latitude selected and processed stating from line 19.

This information contains the completed set of data for a creation of a diagram wave number - frequency

2.2.4   Producing he estimation of rotation velocity

It is necessary to do the following calculations processing the data from output file independently. For each latitude and each maximum of spectral density of wave number spectrum it is necessary to select spectral density maximums of a frequency spectrum which have negative values of frequency. It corresponds to the wave motion in the direction of rotation of the Sun. this data is in file lines describing maximums of frequency spectrum. In the example in the part ``Output set of data" these are line 24, 25 and similar to them. For a wave number 0.043 frequency maximums are: -0.39 -0.23 -0.03 0.17 0.3 0.45 0.130258 0.137045 0.221468 0.128913 0.130603 0.129564

In the first line frequencies are shown and in the second one the values of spectral density are shown. Only three the first frequencies are selected among the frequencies for the analysis. They have negative values: -0.39, -0.23, -0.03. The maximum value of spectral density among these frequencies is a peak with -0.03. This peak is to be selected for the analysis of a dispersion curve. There can be some exceptions from this rule if there appear a maximum of a frequency spectrum in the negative side of a range which has a bit less value of the spectral density than the major maximum, but it fits to the dispersion curve better. This situation is possible because we do not analyze meridian components of waves. So under some conditions it can turn out that the major maximum refers to a wave having a considerable meridian component of a wave number. For a wave number 0.082 the list of frequencies is : -0.45 -0.15 0.04 0.21 0.36 0.132586 0.179725 0.152298 0.13264 0.131843.

In this case there are only two suitable frequency peaks: -0.45 and -0.15. Only one (-0.15) is chosen and it has the maximum value of spectral density 0.179725.

When looking through lists of frequency peaks for each wave number at a given latitude one can form sequences of pair numbers (k_j, f_j), $j = 1,\ldots,M$. Here k_j is a wave number and f_j is a frequency of a major peak. For such set of pair numbers it is possible to make an estimation of phase velocity C at given latitude with the help of linear regression method in accordance to the model:

$$k_j = Cf_j + k_0.$$

With that phase velocity is estimated by formulae given in the chapter Linear regression:

$$C = \frac{\sum_{j=1}^{M} k_j f_j - \overline{k}\,\overline{f}}{\sum_{j=1}^{M} f_j^2 - \overline{f}^2}$$

Here $\overline{k}, \overline{f}$ are the average values of wave number and frequency.

Such estimation can be realized for each latitude separately. As the result a set of pair numbers appear:

$$(C_n, \Theta_n), \ n = -l\ldots,l.$$

Here C_n is an estimation of phase velocity at latitude with number n and Θ_n is the latitude where the phase velocity estimated.

After the formation of such set of estimations a model of differential rotation should be created. The model should be chosen either in a standard form:

$$V(\Theta) = \sin\Theta(B + A\sin^2\Theta).$$

or in any other form corresponding to a definite model of differential model. In case of a standard model coefficients A, B are estimated with the help of least-squares method.

Representation of results

The results of an independent estimation of differential rotation parameters should be represented in a form of phase velocity estimation for each latitude placed in a table and errors of these estimations shown. Parameters of a differential rotation curve in a used model should be represented independently and estimated with the help of least squares method with errors shown.

Task.

1. Choose a sequence of 5-6 images from a database of solar magnetograms received from satellites SOHO (SDO or Hinode).

2. Use the program Cosmophysics and perform a sequence of calculations to obtain a set of data on the distribution of phase velocities by latitude.

3. Graph the dependence of phase velocity of the latitude. Explain the result.

4. Produce a model of the differential rotation of the Sun. Explain the result.

5. Do the same calculations for another time period and compare obtained results.

2.2.5 Charged particles motion in the interplanetary magnetic field. Parker's spiral

Solar cosmic rays (SCR) and solar wind are one of the most important elements of the cosmic space near the Earth. Solar wind is a flow of charged particles with relatively low energy. This flow continuously flows from the solar corona. The solar magnetic field is trapped into solar wind and is transported with it. This makes a so-called Parker's spiral, representing magnetic lines of force bend in a spiral form, which obeys the Archimedes formula:

$$r = \frac{u}{\Omega}(\varphi - \varphi_0) + r_0,$$

here φ_0 is the initial angle of the spiral at the distance r_0 from the Sun, u is the speed of solar wind, Ω is angular velocity of the Sun, r and φ are polar coordinates in ecliptic plane with the center on the Sun which correspond to the magnetic field observation position. Unlike the solar wind more energetic but rare particles of SCR follow the magnetic lines of force, i.e. they move along the Parker's spiral. Moreover, is the solar wind constantly ``blows'' from the Sun, SCR flows grow during solar flares. That is why effects of Parker's spiral formation and SCR motion along it could be observed simultaneously during solar flares. This work is devoted to the exploration of these two parts of the whole charged particles flow ejected by the Sun during solar flares.

Task.

I. Using the known speed of the solar wind registered in the near-Earth space find: (www.spaceweather.com):

1. Its radial component and the direction of arrival to the Earth (angle from the direction to the

Sun).

2. Propagation time from the Sun to the Earth of the registered solar wind.

3. Define the source of the arrived registered solar wind on the Sun.

4. Denote the assumptions taken in the work. Estimate their influence on the obtained result.

II. Knowing the time and coordinates of the solar flare (the information can be taken from the same site):

1. Find the time of the beginning of geomagnetic storm in dependence on the radial propagation speed of the ejected flare matter;

2. Graph the dependence of the registered solar wind speed on the time of its registration in the near-Earth space.

3. Denote the assumptions taken in the work. Estimate their influence on the obtained result.

III. Knowing the moment of the beginning of a strong geomagnetic storm and the speed of the solar wind by that moment (the information can be taken from the same site):

1. Estimate coordinates of the solar flare position able to initiate this storm;

2. Estimate the time of that solar flare.

3. Denote the assumptions taken in the work. Estimate their influence on the obtained result

IV. Look though the solar x-ray radiation monitoring data for previous three days before the time of the beginning of the geomagnetic storm pointed out by a teacher and then single out the moments corresponding to sharp growth of x-ray radiation flux (by factors of ten). The information about the integral x-ray radiation flux from the solar disk obtained by a satellite GOES and placed on site: http://sec.noaa.gov/Data/goes.html.

1. For each burst of x-ray radiation calculate radial velocity of the corresponding ejection of solar matter.

2. According to a model of solar matter motion along the Archimedes' spiral find the area on the Sun and the flare able to cause the storm.

3. Plot the found position of the map of the Sun. If possible find out whether this area was active at the moment and was a flare developing in that area. (archive of a site www.spaceweather.com can be useful) .

For the completion of this task it is recommended that materials corresponding to low solar activity are used. It is possible to use satellite date concerning the arrival of solar plasma flow to the Earth instead of a geomagnetic storm.

3 Mathematical algorithms of data processing

3.1 Method of least squares

3.1.1 The method of least squares

An empirical model most frequently used in the analysis of data is the model in which some parameters, called independent, are related to some parameters, called dependent, with the help of functions of a given form. With that the problem is to calculate unknown parameters of the given function and prove the existence of such dependence. Such problems can be solved with the help of different variants of the method of least squares.

Suppose there is a model of the examined process in the general form:

$$y(t) = \sum_{s=1}^{M} A_s f_s(t) + \delta(t), \tag{3.1}$$

where A_s are the parameters of the model, $f_s(t)$ - strictly given functions of time, $\delta(t)$ - random deviations of the process $y(t)$ from the model. Suppose it is necessary to find out the model (3.1) which should describe a set of measured quantities of the process $y(t)$ at any moment of time $t_i, i = 1, \ldots, N$ in the best way. Denote measured quantities of the process as y_i and δ_i denotes the deviations $\delta(t_i)$ at moments t_i with that it is supposed that deviations δ_i are independent random quantities with zero mean and the same variance. That means the following conditions are true:

$$<\delta_i> = 0; \ <\delta_i \delta_j> = 0, \ i \neq j; \ <\delta_i^2> = \sigma^2. \tag{3.2}$$

Here brackets $<>$ mean statistical average (mathematical expectation) of the corresponding quantities.

The general principle of the method of least squares is that as the best model a model (3.1) should be chosen parameters of which $A_s^{(0)}$ satisfy the condition that the function:

$$\sigma^2(A_1, \ldots, A_M) = \frac{1}{N-1} \sum_{i=1}^{N} \left[y_i - \sum_{s=1}^{M} A_s f_s(t_i) \right]^2 = \sigma^2$$

reaches minimum with $A_s = A_s^{(0)}$. This problem can be reduced to the solution of a set of linear equations in the following common form:

$$\frac{\partial \sigma^2(A_1, \ldots, A_M)}{\partial A_s} = 0, \ A_s = 0, \ldots, M. \tag{3.3}$$

Calculating derivatives we obtain:

$$\frac{\partial \sigma^2}{\partial A_k} = -2 \sum_{i=1}^{N} (y_i - \sum_{s=1}^{M} A_s f_s(t_i)) f_k(t_i) = 0, \ k = 1, \ldots, M.$$

This set of equations can be written as:

$$\sum_{j=1}^{M} [\sum_{i=1}^{N} f_k(t_i) f_j(t_i)] A_j = \sum_{i=1}^{N} f_k(t_i) y_i, \ k = 1, \ldots, M,$$

Or in matrix form:

$$QA = F, \tag{3.4}$$

where $A = (A_1, A_2, \ldots, A_M)$, matrix Q and vector F have the following elements:

$$Q_{kj} = \sum_{i=1}^{N} f_k(t_i) f_j(t_i), \ \ F_k = \sum_{i=1}^{N} y_i f_k(t_i).$$

The solution of (3.4) can be written as:

$$A = Q^{-1} F.$$

Or in component form this relation looks like:

$$A_j = \sum_{k=1}^{M} R_{jk} (\sum_{i=1}^{N} y_i f_k(i)), \ j = 1, \ldots, M, \tag{3.5}$$

here R_{ik} - are matrix elements of matrix $R = Q^{-1}$, inverse to Q.

Calculate error of the regression parameters estimation. To do that, introduce the following determinations and designations. Denote true coefficients of regression as $A_k^{(0)}$ supposing the considered model to take place in reality. Coefficients A_k obtained as the result of calculations and satisfying relations (3.5) are to be called estimations of true regression coefficients. Substitute

expressions for y_i from (3.1) into (3.5) supposing true regression coefficients $A_k^{(0)}$ to be in that expressions. As the result we obtain:

$$A_j = \sum_{k=1}^{M} R_{jk} \left(\sum_{i=1}^{N} (\sum_{s=1}^{M} A_s^{(0)} f_s(t_i) + \delta_i) f_k(t_i) \right), \ j = 1, \ldots, M,$$

Transforming this relation we obtain:

$$A_j = \sum_{k=1}^{M} \sum_{s=1}^{M} R_{jk} Q_{ks} A_s^{(0)} + \sum_{i=1}^{N} \delta_i \sum_{k=1}^{M} R_{jk} f_k(t_i), \ j = 1, \ldots, M,$$

As far as matrixes Q and R are reverse to each other, i.e.:

$$\sum_{k=1}^{M} \sum_{s=1}^{M} R_{jk} Q_{ks} = \delta_{js},$$

where $\delta_{kk} = 1, k = 1, \ldots, M$ and $\delta_{ks} = 0, k \neq s$, we finally obtain:

$$A_j = A_j^{(0)} + \sum_{i=1}^{N} \delta_i \sum_{k=1}^{M} R_{jk} f_k(t_i), \ j = 1, \ldots, M. \tag{3.6}$$

According to the condition (3.2) we find:

$$< A_j > = A_j^{(0)}.$$

The latter condition means that the estimation of A_j is **not displaced**. Therefore:

$$\sigma_{A_j}^2 = < (A_j - A_j^{(0)})^2 > = \sigma^2 R_{jj}.$$

These equations can be used for calculations in general case but it is more useful to consider specific models which are mentioned in this cosmophysical practical work.

3.1.2 Linear regression

Estimation of regression coefficients

In the majority of laboratory works it is necessary to use the method of least squares to estimate inclination of the time dependence of different orbital parameters and parameters of satellites motion. This is the simplest variant of the model (3.1).

Suppose the physical parameter y to depend on any other parameter t (for example the time) in the following way:

$$y(t) = At + B + \delta(t),$$

where A and B are the parameters of the model. Following the general instruction we obtain:

$$\frac{\partial \sigma^2(A, B)}{\partial A} = -2 \sum_{i=1}^{N} (y_i - At_i - B) t_i = 0, \tag{3.7}$$

$$\frac{\partial \sigma^2(A, B)}{\partial B} = -2 \sum_{i=1}^{N} (y_i - At_i - B) = 0. \tag{3.8}$$

Introduce the following designations:

$$\bar{Y} = \frac{1}{N} \sum_{i=1}^{N} y_i, \ \bar{T} = \frac{1}{N} \sum_{i=1}^{N} t_i,$$

$$\sigma_T^2 = \frac{1}{N} \sum_{i=1}^{N} t_i^2, \ C_{YT} = \frac{1}{N} \sum_{i=1}^{N} y_i t_i.$$

Then equations (3.7) and (3.8) can be written as:

$$\sigma_T^2 A + \bar{T}B = C_{YT},$$
$$\bar{T}A + B = \bar{Y}.$$

Solving the set of two linear algebraic equations we find:

$$A = \frac{C_{YT} - \bar{Y}\bar{T}}{\sigma_T^2 - \bar{T}^2},$$

$$B = \bar{Y} - A\bar{T}.$$

Such values of the model parameters are accepted as the best ones according to the method of least squares. With the help of such calculations the estimation of variation speed of the ascending node, perigee argument and the speed of decrease of total energy is realized in problems of this practical work.

Calculation of the error of linear regression coefficients estimation

To calculate the error of estimation of A and B coefficients represent their solution as:

$$A = \frac{1}{\sigma_T^2 - \bar{T}^2} \frac{1}{N} \sum_{i=1}^{N} y_i t_i,$$

$$B = \frac{1}{N} \sum_{i=1}^{N} y_i - A\bar{T}.$$

Denote as A_0 and B_0 the true values of model coefficients which unlike A and B are to be called estimations A_0 and B_0. The relation between A_0, B_0 and A, B can be obtained if we substitute y_i by values from the model taking into account random deviates $\delta_i = \delta(t_i)$ in the expression for A and B. As the result we obtain:

$$A = A_0 + \frac{1}{\sigma_T^2 - \bar{T}^2} \frac{1}{N} \sum_{i=1}^{N} \delta_i t_i,$$

$$B = B_0 + \frac{1}{N} \sum_{i=1}^{N} \delta_i$$

It is supposed that:

$$\bar{\delta} = \frac{1}{N} \sum_{i=1}^{N} \delta_i = 0, \sigma^2 = \frac{1}{N-1} \sum_{i=1}^{N} \delta_i^2,$$

And correlations between δ_i and δ_j with $i \neq j$ are equal to 0, i.e.:

$$< \delta_i \delta_j > = 0.$$

In this case it is possible to calculate the average and variance of the estimates A and B. Calculations yield the following:

$$< A > = A_0, \quad < B > = B_0, \tag{3.9}$$

$$\sigma_A^2 = D_A = \frac{\sigma^2}{N(\sigma_T^2 - \bar{T}^2)}, \quad \sigma_B^2 = D_B = \frac{\sigma^2}{N}. \tag{3.10}$$

Quantities σ_A and σ_B are the errors of the calculation of linear regression coefficients.

3.1.3 Square regression

In a similar way it is possible to make the estimation of square regression parameters. The

square regression model has the following form:
$$y(t) = At^2 + Bt + C + \delta(t).$$
Equations (3.3) in this case look like:
$$AT_4 + BT_3 + CT_2 = Y_2,$$
$$AT_3 + BT_2 + CT_1 = Y_1,$$
$$C = Y_0 - AT_2 - BT_1.$$
Excluding C we come to:
$$M_1 A + M_0 B = F_1, \quad M_0 A + M_2 B = F_2, \tag{3.11}$$
where
$$M_1 = T_4 - T_2^2, \quad M_2 = T_2 - T_1^2, \quad M_0 = T_3 - T_1 T_2,$$
$$F_1 = Y_2 - T_2 Y_0, \quad F_2 = Y_1 - T_1 Y_0,$$
And quantities T_m and Y_n are determined as:
$$T_m = \frac{1}{N} \sum_{i=1}^{N} t_i^m, \ m = 1,2,3,4, \ Y_n = \frac{1}{N} \sum_{i=1}^{N} y_i t_i^n, \ n = 0,1,2.$$
The solution of relations (3.11) has the following form:
$$A = \frac{M_2 F_1 - M_0 F_2}{M_1 M_2 - M_0^2}, \quad B = \frac{-M_0 F_1 + M_1 F_2}{M_1 M_2 - M_0^2}. \tag{3.12}$$
Parameter C can be found from the relation $C = Y_0 - AT_2 - BT_1$.

This model can be used to estimate the deviation of the longitude of ascending node from secular motion.

3.1.4 Model of a harmonic process

In applied problems of analysis of different physical and other processes it is often necessary to single out harmonic processes and estimate their basic parameters. A model of one-dimension harmonic process $y(t)$ with the natural frequency ω has the following form:
$$y(t) = A\cos \omega t + B\sin \omega t + C + \delta(t). \tag{3.13}$$
According to the general principle of the method of least squares the condition of minimum of function σ has the form:
$$\frac{\partial \sigma^2}{\partial A} = A(1 - C_2) - S_2 B + 2S_1 C - 2S_y = 0,$$
$$\frac{\partial \sigma^2}{\partial B} = B(1 + C_2) - S_2 A + 2C_1 C - 2S_y = 0,$$
$$\frac{\partial \sigma^2}{\partial C} = C + AS_1 + BC_1 - \bar{Y} = 0,$$
where
$$C_2 = \frac{1}{N} \sum_{i=1}^{N} \cos 2\omega t_i, \ S_2 = \frac{1}{N} \sum_{i=1}^{N} \sin 2\omega t_i,$$
$$C_1 = \frac{1}{N} \sum_{i=1}^{N} \cos \omega t_i, \ S_1 = \frac{1}{N} \sum_{i=1}^{N} \sin \omega t_i,$$

$$\bar{Y} = \frac{1}{N} \sum_{i=1}^{N} y_i, \quad S_y = \frac{1}{N} \sum_{i=1}^{N} y_i \sin \omega t_i, \quad C_y = \frac{1}{N} \sum_{i=1}^{N} y_i \cos \omega t_i.$$

After the exclusion of C the equations for harmonic's amplitudes A and B take again the form of equations (3.11) where coefficients should be calculated from:

$$M_1 = 1 - C_2 - 2S_1^2, \quad M_2 = 1 + C_2 - 2C_1^2,$$
$$M_0 = -S_2 - 2C_1 S_1,$$
$$F_1 = 2[S_y - S_1 \bar{Y}], \quad F_2 = 2[C_y - C_1 \bar{Y}].$$

The solution of (3.11) will have the same form (3.12). Parameter C should be calculated as:

$$C = -AS_1 - BC_1 - \bar{Y}.$$

The total amplitude of a harmonic and the estimation of the initial phase have the following form:

$$A_0 = \sqrt{A^2 + B^2}, \quad \phi = \frac{B}{A}.$$

This model can be used to estimate parameters of secular motion, perigee argument and mean anomaly estimation.

3.1.5   Calculation of measurement error of complicated parameters

One of the targets of this practical work is the problem of errors calculation of quantities which are the functions of quantities measured directly in the experiment. Such quantities are to be called complicated. In problems concerning the measurement of parameter J_2, parameters measured directly in the experiment are the parameters of orbital motion of a satellite: perigee argument, longitude of ascending node etc. Parameter J_2 is a function of these parameters.

Suppose a measured parameter p to be some function of directly measured quantities X_i, $i = 1, \dots, L$, i.e.:

$$p = P(X_1, X_2, \dots, X_L).$$

Then suppose these directly measured quantities X_i to be characterized by the estimation of mean values $\bar{X}_i$ and the total statistical and instrument inaccuracy equal to σ_i, i.e. σ_i^2 is a variance of scattering generated by uncontrolled outer reasons and inaccuracy of an instrument. Then as a p parameter estimation a value is chosen:

$$\bar{p} = P(\bar{X}_1, \bar{X}_2, \dots, \bar{X}_L),$$

In case of independence of random variances of single quantities X_i the absolute error of p parameter measurement is determined by:

$$\sigma_p = \sqrt{\left| \frac{\partial P}{\partial X_1} \right|^2 \cdot \sigma_1^2 + \left| \frac{\partial P}{\partial X_2} \right|^2 \cdot \sigma_2^2 + \cdots + \left| \frac{\partial P}{\partial X_L} \right|^2 \cdot \sigma_L^2} \Bigg|_{X_1 = \bar{X}_1, X_2 = \bar{X}_2, \dots, X_L = \bar{X}_L}. \qquad (3.14)$$

Relative error is calculated from:

$$\frac{\sigma_p}{\bar{p}} = \sqrt{\left| \frac{\partial \ln P}{\partial X_1} \right|^2 \cdot \sigma_1^2 + \left| \frac{\partial \ln P}{\partial X_2} \right|^2 \cdot \sigma_2^2 + \cdots + \left| \frac{\partial \ln P}{\partial X_L} \right|^2 \cdot \sigma_L^2} \Bigg|_{X_1 = \bar{X}_1, X_2 = \bar{X}_2, \dots, X_L = \bar{X}_L}.$$

Calculation of measurement errors for J_2 parameter

As the example let's consider the calculation of measurement error for parameter J_2 by the data on longitude of ascending node. The corresponding equation for the estimation of J_2 is:

$$\frac{d\Omega}{dt} = \frac{3}{2}\sqrt{\frac{GM}{a^3}}J_2\left(\frac{R_E}{p}\right)^2\frac{\cos i}{(1-e^2)^2}. \tag{3.15}$$

Notice that directly measured parameters are included: i - inclination, p - focal parameter, n - mean motion and the inclination of the linear regression curve R_{anl}. The Earth radius is supposed to be a constant value measured with a perfect accuracy. The same is about the eccentricity e which is supposed to be zero. That refers to a circular orbit. Notice that in the considered approximation quantities i, p, n are supposed to be constant. But analysis of the data shows that within a period of time when parameters of linear regression are estimated all these parameters evolve considerably especially if rather long periods of time are considered. So in this approximation these variations of parameters must be considered as random (unpredictable) ones and to estimate parameter J_2 by the relation (3.15) the average values of the given period of time should be taken. For small orbit eccentricities $e \sim 0$, which are examined in this practical work, quantities p and n are dependent and are connected by the relation:

$$p = \left(\frac{GM}{n^2}\right)^{1/3}.$$

So to calculate the error of J_2 only n and i variance should be substituted into (3.14)). (3.15) also includes parameter R_{anl} of the linear regression of a curve of the longitude of ascending node secular variation. To estimate its error it is necessary to use (3.10) for the estimation of A parameter of the linear regression.

As the result we obtain the following relation for the estimation of the relative error of J_2:

$$\frac{\sigma_{J_2}}{\bar{J}_2} = \sqrt{\frac{\sigma_{R_{anl}}^2}{R_{anl}^2} + \frac{49}{9}\frac{\sigma_n^2}{n^2} + \frac{\sigma_i^2\sin^2 i}{\cos^4 i}}. \tag{3.16}$$

It follows that i variations can lead to considerable errors in measurement of J_2 for satellites on near-polar orbits when $\cos i \ll 1$.

For perigee argument error can be calculated as:

$$\frac{\sigma_{J_2}}{\bar{J}_2} = \sqrt{\frac{\sigma_{R_{anl}}^2}{R_{anl}^2} + \frac{49}{9}\frac{\sigma_n^2}{n^2} + 25\frac{\sigma_i^2\cos^2 i\sin^2 i}{(1-5\cos^2 i)^4}}, \tag{3.17}$$

And for a mean anomaly:

$$\frac{\sigma_{J_2}}{\bar{J}_2} = \sqrt{\frac{\sigma_{R_{anl}}^2}{R_{anl}^2} + \frac{49}{9}\frac{\sigma_n^2}{n^2} + 9\frac{\sigma_i^2\cos^2 i\sin^2 i}{(1-3\cos^2 i)^4}}, \tag{3.18}$$

Here σ_i and σ_n are the errors of inclination and mean motion including statistical variance of measurement of these quantities, $\sigma_{R_{anl}}$, $\sigma_{R_{arp}}$, $\sigma_{R_{aan}}$ - errors of the method of linear regression for corresponding parameters.

3.1.6 Time series filtration

Time series filtration is understood as a linear transformation allowing change of Fourier

spectrum of this series deleting a part of harmonic components. Suppose there is a discrete time series of measurements $u_0, u_1, u_2, \ldots, u_N$ of a parameter $u(t)$ taken in equal periods of time Δt:

$$u_k = u(t_k), \ \ t_k = k\Delta t, \ k = 0,1,2,\ldots,N.$$

In this case the linear smoothing (low frequency transmission band) filtration is understood as the transformation:

$$U_k = \sum_{j=-P}^{P} W_j u_{k-j}, \ \ k = P,\ldots,N-P. \tag{3.19}$$

Here real not negative numbers $W_j \geq 0, \ j = -P,\ldots,P$ are called filter coefficients and integer number P is called half-width of the filter window. Numbers W_j must satisfy normalizing conditions:

$$\sum_{j=-P}^{P} W_j = 1,$$

And symmetry conditions:

$$W_j = W_{-j}, j = -P,\ldots,P.$$

Except smoothing and low-frequency filtration a high-frequency filtration is also used. It allows to delete trends and band filtration. High-frequency filtration is a linear transformation including the transformation (3.19) and the consequent subtraction of the obtained series from the initial:

$$V_k = u_k - U_k, \ \ k = -P,\ldots,P. \tag{3.20}$$

Here numbers U_k are determined by (3.19). Band filtration represents the application of sequential transformations of low-frequency filtration with a window half-width P and high-frequency filtration with a window half-width $Q > P$. in case of high-frequency filtration the obtained series contains frequency components higher than some characteristic cut-off frequency of a filter and in case of band filtration - in some limited band of frequencies.

Using filters it is necessary to take into account that their usage is connected with the decrease of the initial series length by the value of the window width $2P$. It is clear from the determinations of their linear transformations. It is very important when using high-frequency filters, which have a big window half-width that leads to a considerable decrease of the initial series length and as the result to the loss of accuracy and the reliability of statistical estimation based on them.

Depending on the choice of filter coefficients W_k there are a lot of different types of smoothing filters which determine quantity of different ways of blanking of high-frequency components of the incoming signal spectrum. We examine only three of them. Other types are described in different books devoted to data processing. [13, 14, 15]). Notice that for the purpose of problem 3 there is no difference which filter to apply. Choice of the window half-width plays an important role for the obtaining quite reliable focal parameter derivative values. Choice of the half-width is realized during the work with the certain time series.

Rectangular filter

Rectangular filter is determined by the choice of coefficients according to the following rule:

$$W_k = w_0, \ \ k = -P,\ldots,P. \tag{3.21}$$

Basing on the normalizing condition we obtain that the number w_0 must be related to the window half-width as:

$$w_0 = \frac{1}{2P+1}.$$

Curves of filter weight factor are shown in Tab. 2.1 (a,b,c).

Cosine filter (Tukie)

Cosine filter is specified by the relation

$$W_k = \frac{1}{2P}\left(1+\cos(\frac{\pi k}{P})\right), \ k = -P,\ldots,P. \qquad (3.22)$$

Triangular filter

Triangular filter is specified by the relation:

$$W_k = \frac{1}{P^2}(2P-|k-P|), \ k = -P,\ldots,P. \qquad (3.23)$$

3.1.7 Piecewise-linear regression and piecewise-cubic interpolation with smoothing

Although the usage of filters in order to obtain smoothed estimation of time dependence of a focal parameter is the simplest way of this problem solution, it is interesting to consider other types of smoothing. The main problem of smoothed estimation obtaining with the help of smoothing filters is mentioned above decrease of the filtered series length in comparison to the initial one. Method of piecewise-linear regression and smoothing cubic spline described in monograph [12] are methods allowing to escape this problem. Consider them separately [12][9]). Consider them separately.

Piece-wise regression

In problem 3 the main problem is the obtaining of a stable estimation of time variation speed of a focal parameter. Method of piecewise -linear regression enables to solve this problem with the help of least-square method used with a linear model applied to separate small pieces of a time series, in our case to focal parameter measurements. To do that the whole series should be divided into some number K of segments containing N_k, $k = 1,\ldots,K$ points so that the total number of points of N series is equal to the sum of segment points:

$$N = \sum_{k=1}^{K} N_k.$$

Numbers N_k can differ, but the simplest way of the choice is their equality $N_k = [N/K]$, $k = 1,\ldots,K-1, N_K = N - (K-1)[N/K]$, where $[f]$ denotes an integer part of a real number f. After the definition of segment edges $[t_{k-1}, t_k]$, $k = 1,\ldots,K$ linear regression parameters A_k, B_k should be calculated for measured parameter values (focal parameter) in each segment:

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[9]G.I. Marchuk. Methods of computational mathematics. M.: Nauka, 1977. 454 P.

$$u = A_k t + B_k, \ t \in [t_{k-1}, t_k], \ k = 1, \ldots, K.$$

Coefficient A_k is an estimation of the variation speed of parameter u in each segment. Attributing this value to all points of a segment we obtain piecewise constant approximation of parameter u variation speed. In this case the degree of ``smoothing'' depends on the number of segments and their length. The more segments are chosen the shorter their length on the average and the less become smoothing and stability or reliability of the derivative estimation decrease. So the number of segments should be relatively small. The optimal number of segments is selected from the experiment for each series separately. That is a considerable disadvantage of this method. But the simplicity of its realization is its relative advantage.

Cubic splines

To escape difficulties related both to filtration (decrease of a series length) and piecewise linear regression it is useful to apply methods of spline interpolation. One of the simplest spline interpolation methods is a cubic splines method and its modification - method of smoothing cubic spline-function.

Suppose in a segment $[a,b]$ of a real line there is an array $a = x_0 < x_1 < x_2 < \ldots < x_N = b$ in nodes of which values of a function $u_k = u(x_k)$ are specified. The cubic spline functions method realizes the idea of interpolation of $u(x)$ function values into intermediate points of an array with the help of flexible elastic ruler passing all the points $P_k = (x_k, u_k)$ of a plane x, u. British draughtsmen of the beginning of the XX century called such ruler spline.

Elastic ruler curved under the law $g = g(x)$ in such way to pass all given points $P_k = (x_k, u_k)$ delivers a minimum to a functional of a thin band curvature. This functional can be written as:

$$E = \int_a^b (g'')^2 dx, \tag{3.24}$$

where

$$g'' = \frac{d^2 g}{dx^2}.$$

It is necessary find the minimum of functional (3.24) under the condition, that it passes all points P_k, in order to find the shape of a ruler $g(x)$ called spline-function. As the result with the help of Lagrangian coefficients we come to a problem of an absolute extremum of a functional:

$$E_1 = \int_a^b (g'')^2 dx + \sum_{k=1}^K \lambda_k (g(x_k) - u_k),$$

where λ_k are Lagrangian coefficients.

The solution of this problem is equal to a function $g(x)$ representing a cubic polynomial of the following form in each segment $[x_{k-1}, x_k]$:

$$g_k(x) = m_k x^3 + n_k x^2 + p_k x + q_k, \ x \in [x_{k-1}, x_k]. \tag{3.25}$$

With that the conditions should be satisfied at the edges of segments:

$$g_{k''}(x_k) = g_{k+1}''(x_k), \ g_{k'}(x_k) = g_{k+1}'(x_k), \ g_k(x_k) = g_{k+1}(x_k), \tag{3.26}$$

i.e. in general $g(x)$ is a function of class $Ñ^2$, i.e. it has continuous derivatives up to the second order.

The solution of that problem is described in [12]. We just show the final result of this problem

solution.

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