SAMARA STATE AEROSPACE UNIVERSITY

Exercises for the Position computations course

Samara 2014

Author: Kai Borre

Exercises for the Position computations course / Samara State Aerospace University.
Samara 2014

Recommended for the students studying programs 11.04.01 «GNSS receivers. Hardware and software» and 03.04.01 «Algorithms and software»

This edition contains exercises as announced by the teacher in each lecture.

Printed according the decision of editorial committee of Samara State Aerospace University.

Contents

Introduction

Each laboratory session includes one or more exercises as announced by the teacher in each lecture.

In addition to the laboratory sessions, each student performs an individual home work. This work consists in finding solutions to a defined set of these exercises. The solutions are collected in a small written report. The report is delivered not later than two weeks after end of the course.

Phase observations

The following ten exercises are related to the M-file easy15. Start by running that file and you will be able to answer the questions:

1. Identify the PRN of the reference satellite
2. Part of the code uses a gain matrix K. What is the range of the entries of K and what is their dimension?
3. What is the variation of the master position over the first ten epochs? Hint: `diff(master_pos(:,1:10),1,2)`
4. What are the actual values of the ambiguities `amb`?
5. Open Figure 2. Do you believe the estimated ambiguities are the correct ones?
6. Describe the differencing matrix D. Explain the position of the columns of mere $+1$s and -1s
7. Explain the role of the matrix $\Sigma = DD^{\mathrm{T}}$
8. Compute `norm(A(:,1:3))`. Does this result surpise you? Next compute `norm((Xk_ECF-X_j)/rhok_j)` and `norm((Xl_ECF-X_j)/rhol_j)`. Compare this result with the equation below (10.16) in Borre & Strang. We are dealing with the difference of two unit vectors.
9. Given

$$\begin{aligned} \Phi_1 &= \rho^* - I + \lambda_1 N_1 \\ \Phi_2 &= \rho^* - \alpha I + \lambda_2 N_2 \end{aligned} \tag{1}$$

   where $\alpha = (f_1/f_2)^2$. Eliminate ρ^* and derive an expression for I. This is (10.21) in Borre & Strang.
10. Eliminate I from (1) and obtain an expression for ρ^*. This combination is called the *ionosphere free combination* of the phase observations, see (10.23) in Borre & Strang.

Mathematical models

11. We want to investigate the correlation between the ambiguities N_1 and N_2. They are determined from the linear equation $Ax = b$ or

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & \lambda_1 & 0 \\ 1 & \alpha & 0 & 0 \\ 1 & -\alpha & 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} \rho \\ I \\ N_1 \\ N_2 \end{bmatrix} = \begin{bmatrix} P_1 \\ \Phi_1 \\ P_2 \\ \Phi_2 \end{bmatrix}.$$

The constants α, λ_1, and λ_2 are defined as follows

$$\begin{aligned} c_0 &= 299792458 \\ f_1 &= 154 \times 10.23 \times 10^6 \\ f_2 &= 120 \times 10.23 \times 10^6 \\ \lambda_1 &= c_0/f_1 \\ \lambda_2 &= c_0/f_2 \\ \alpha &= (f_1/f_2)^2. \end{aligned}$$

A realistic weight matrix is

$$C = \begin{bmatrix} 1/0.3^2 & & & \\ & 1/0.003^2 & & \\ & & 1/0.3^2 & \\ & & & 1/0.003^2 \end{bmatrix}.$$

Now compute the covariance matrix for the vector x of unknowns $\Sigma_x = (A^{\mathrm{T}}CA)^{-1}$. The lower right 2 by 2 block matrix is the covariance matrix for N_1 and N_2.

Compute eigenvalues and eigenvectors of this matrix and sketch the confidence ellipse.

Hint: You may use calls like

```
Sigma_N = Sigma_x(3:4,3:4);
[a,v] = eig(Sigma_N)
support(Sigma_N)
```

Ambiguity resolution

12. Run easy12. The curve in Figures 2–9 intersects the x-axis at various values for x, most often circa $c_0 = 125$ s. For time spans shorter than c_0 there is a positive correlation between I-values belonging to different time values. While for time intervals larger than c_0 there is a negative correlation, confer the definition of autocorrelation in (5.12) in Borre & Strang, and page 119 at top.

13. A reset of the receiver clock affects all pseudoranges at that very epoch and all subsequent ones. Does a clock reset of ± 1 ms influence the position computation? If not, why so?

14. A clock reset causes a special problem for a receiver that observes both pseudoranges and carrier phases. There are several ways to handle the situation. Describe one or two of those.

Estimating differential corrections at a base station

15. Run the M-script easy18. The variable pos(4,:) describes the clock offset in units of meters. Plot the variable and indicate the circa value of the clock offset in seconds.

16. Run the M-script easy18. The variable diff(pos(4,:)) has a circa value of 67 meters per second or circa 200 ns/s. How many epochs are there between clock resets of 1 ms?

Real time kinematic

17. The M-script rtk1 contains two procedures for estimating the ambiguities. One method called goad which is called on line 64, and another method called teunissen on line 65. Run the script first with the goad algorithm and next with the teunissen algorithm and compare the two estimates for the baseline. Why do we have a discrepancy of about 0.784 meter?

Educational edition

Exercises for the Position computations course

Leaner’s guide

Author: Kai Borre

SAMARA STATE AEROSPACE UNIVERSITY (SSAU)