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Кафедра иностранных языков

Л.С. Бабич

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Рецензент канд. пед. наук, доц. Л.В.Сологуб

Бабич Л.С.

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Основная цель пособия – формирование навыков чтения, перевода и реферирования оригинальной англоязычной литературы по специальности, а также умения вести беседу на профессиональные темы.

Пособие состоит из 9 тематических уроков, включающих предтекстовые задания и лексико-грамматический комментарий и послетекстовые упражнения, направленные на развитие устной речи.

Раздел дополнительного чтения содержит тексты экономического характера с набором вопросов, ответы на которые предполагают раскрытие и творческое осмысление проблемы. Не исключено использование отдельных текстов данного раздела и для совершенствования навыков перевода.

Тексты пособия заимствованы из аутентичных источников.

Пособие снабжено греческим алфавитом и примерами чтения математических символов и формул по-английски.

Предназначено для студентов II курса дневного отделения специальности «Математические методы в экономике». Некоторые его разделы могут быть использованы и для студентов специальности «Экономика».

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UNIT 1

TEXT A

Ex. 1. Read the text and find all the verbs in the Simple Active and Simple Passive Tenses:

I. ECONOMICS AS A SCIENCE

The dictionary defines economics as "the study of wealth and the consumption of goods and services in a society".

Economics is a broad subject. It's a social science, because it studies the social problem of choice from a scientific viewpoint. Economics studies how individuals, firms, governments and other organizations within our society make choices, and how those choices determine the way the resources of society are used. The economy's limited resources include its land, labour, machines, oil and others.

The detailed study of product, labour and capital markets is called microeconomics ("micro" is derived from the Greek word meaning "small"). It is focused on the behaviour of the units – the firms, households, and individuals – that make up the economy. It's concerned with how the individual units make decisions and what affects those decisions.

By contrast, macroeconomics ("macro" comes from the Greek word "large") looks at the behaviour of the economy as a whole, in particular the behaviour of such aggregate measures as overall rates of unemployment, inflation, economic growth, and the balance of trade.

Microeconomics is the bottom-up view of the economy; macroeconomics is the top-down view.

Economists use models to study how the economy works and to make predictions about what will happen if something is changed. A model can be expressed in words or equations. The economists' basic model has three components: assumption about how consumers behave; assumption about how firms behave and assumption about the markets in which these consumers and firms interact.

Economists are frequently called upon to make judgements on matters of public policy: should the government reduce the deficit; should inflation be reduced; if so, how?

While rational choices involve balancing of costs and benefits, economists spend more time discussing costs than benefits. This is largely because individuals and firms often see clearly the benefits of each alternative; where they make mistakes is in evaluating the costs. So, it is of prime importance to think systematically about costs.

Ex. 2. Form “-tion”, “-ation” nouns from the following verbs and translate them:

- | | |
|---------------|----------------|
| ▪ to limit | ▪ to assume |
| ▪ to interact | ▪ to inflate |
| ▪ to produce | ▪ to equate |
| ▪ to consume | ▪ to organize |
| ▪ to reduce | ▪ to determine |

Ex. 3. Make “-ment” nouns from the following verbs and translate them into Russian:

- | | |
|-------------|--------------|
| ▪ to govern | ▪ to develop |
| ▪ to judge | ▪ to state |
| ▪ to employ | ▪ to involve |
| ▪ to manage | |

Ex. 4. Think of all possible sentences containing the words and word combinations below and write them down:

- | | |
|---------------------|-----------------------------|
| ▪ to make choices | ▪ components |
| ▪ limited resources | ▪ to make predictions about |
| ▪ microeconomics | ▪ wealth |
| ▪ macroeconomics | ▪ to make judgements on |
| ▪ equations | |

Ex. 5. Give English equivalents for the following Russian phrases:

- | | |
|---|-------------------------------|
| ▪ принимать решения | ▪ потребление товаров и услуг |
| ▪ поведение | ▪ влиять на решения |
| ▪ экономический рост | ▪ выражать в уравнениях |
| ▪ делать прогнозы относительно... | ▪ безработица |
| ▪ взгляд на экономику сверху вниз / снизу вверх | ▪ темпы инфляции |
| ▪ определять то, как... | ▪ доходы/расходы |
| ▪ предположение относительно рынков | ▪ рынок труда |

Ex. 6. Work in pairs. Ask and answer the following questions:

1. What is the fundamental definition of economics?
2. Can you recognize the difference between microeconomics and macroeconomics?
3. What do you understand by “limited resources”?
4. What is the role of mathematical models in economic analysis?
5. Give the three components that form the economists’ basic model.
6. Why is it important in economics to make choice from a scientific viewpoint?

7. What are the essential matters of public policy economists make judgements on?
8. Do you agree that economists must spend more time discussing costs than benefits? Why?

Ex. 7. Be ready to speak on the topic "Economics as a Science".

TEXT B

Ex. 1 Practise the pronunciation of the following words and word combinations and guess their meaning:

- | | |
|-----------------|--------------------|
| ▪ mathematics | ▪ characteristic |
| ▪ mathematician | ▪ symbol |
| ▪ science | ▪ symbolism |
| ▪ scientist | ▪ to specialize in |
| ▪ scientific | ▪ mechanically |
| ▪ alphabet | ▪ fundamental |
| ▪ logic | ▪ efficiency |
| ▪ style | ▪ to confuse |
| ▪ civilized | ▪ compactness |

Ex. 2. Read and translate the text consulting the *Essential Vocabulary* below:

- | | | |
|------------------------------|---|-----------------------------|
| ✓ layman | = | неспециалист |
| ✓ scope | = | пределы, область, сфера |
| ✓ sign | = | знак |
| ✓ in a sense | = | в известном смысле |
| ✓ vocabulary | = | словарный состав |
| ✓ abbreviation | = | сокращение, аббревиатура |
| ✓ to stand for | = | заменять, обозначать |
| ✓ to grasp | = | понять, постичь |
| ✓ to make for | = | служить |
| ✓ precise | = | точный |
| ✓ concise | = | краткий, сжатый |
| ✓ to aim at | = | стремиться к |
| ✓ brevity | = | краткость |
| ✓ to encompass | = | заключать в себе |
| ✓ formal perfection | = | совершенство форм |
| ✓ to have the relation to... | = | иметь отношение к чему-либо |
| ✓ compass | = | циркуль |
| ✓ to encompass | = | заключать в себе |

II. THE LANGUAGE OF MATHEMATICS

Mathematics is involved in everything around us. It has been called "the queen of knowledge". It is a tool which helps man to know how much, how many, how large, how fast, in what direction. Mathematics is, to use a common phrase, "the language of science". Mathematics has been supplying a language, methods and conclusions for science; enabling scientists to predict results; preparing the minds of scientists for new ways of thinking.

But it is useful not only to those who specialize in science. Even a layman must know something about the foundations, the scope and the basic role played by mathematics in our scientific age.

The language of mathematics is known to consist mostly of signs and symbols and, in a sense, it is an unspoken language. It is the same throughout the civilized world, though the people of each country translate it into their own particular spoken language.

Being a language, the language of mathematics has definite vocabulary and grammar rules. Its vocabulary comprises the best known symbols of mathematics: the Arabic numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, the signs of addition, subtraction, multiplication, division, equality and the letters of the alphabets – Greek, Latin, Gothic. It also includes abbreviations of words, which usually have no relation to the thing they stand for. We determine their meaning by definition and use them by common agreement. Grammar rules are the rules of logic.

So, mathematics is more than just a system of numerals. It is also a way of thinking and a form of logical reasoning. Symbolic language is one of the basic characteristics of modern mathematics. By the aid of symbolism mathematicians can make transitions in reasoning almost mechanically by the eye and leave their mind free to grasp the fundamental ideas of the subject matter. This compactness makes for efficiency of thought.

Mathematical language is precise and concise. Its symbolism is essential to distinguish meanings often confused in common speech. Mathematical style aims at brevity and formal perfection.

It is remarkable, because it encompasses much in few words, and most mathematicians find beauty and even poetry in it. The language of mathematics is carefully and purposefully designed by the best brains of all countries and all times.

Ex. 3. Brush up your grammar. Find the Participles (I, II) in the text. State their functions.

Ex. 4. Work in pairs. Ask and answer the following questions:

1. Why is mathematics called the language of science?
2. Why is the language of mathematics universal?
3. Has this language definite vocabulary and grammar rules?
4. What do abbreviations of words mean?
5. How can mathematicians make transitions in reasoning?
6. Do you agree with the statement that the language of mathematics is remarkable?

Ex. 5. Complete the following sentences:

1. Mathematics is to use a common phrase ...
2. The language of mathematics, is the same...
3. The best known symbols of mathematics are ...
4. By the aid of symbolism mathematicians ...
5. Mathematical style...

Ex. 6. Translate into English:

1. Математический язык в известном смысле не разговорный язык, так как это язык знаков и символов.
2. Не существует более простого и универсального языка, чем язык математики.
3. Математика обеспечивает науку языком, новыми методами мышления, выводами, и позволяет ученым прогнозировать результаты.
4. Символы – это сокращения слов, но часто они не имеют никакого отношения к словам, которые они заменяют. Мы определяем их значение по определению.
5. Язык математики точен и краток. Он позволяет делать переходы в рассуждениях почти механически, зрительным образом.

Ex. 7. Speak on the sub-topics:

1. Mathematics is the language of science.
2. The vocabulary of the language of mathematics.
3. The compactness and elegance of mathematical style.

UNIT 2

Ex. 1. Practise the pronunciation of words and word combinations and give their Russian equivalents:

- | | | |
|---------------------|--------------|--------------|
| ▪ public finance | ▪ postulate | ▪ deductive |
| ▪ urban economics | ▪ technique | ▪ relevant |
| ▪ integral calculus | ▪ hypothesis | ▪ exp'licit |
| ▪ in the sense | ▪ analysis | ▪ imp'licit |
| ▪ matrix algebra | ▪ dispute | ▪ to utilize |

Ex. 2. Pay attention to the meaning of the following phrasal verbs and memorize them:

- | | | |
|---------------------|---|---|
| ▪ to draw upon | = | обращаться к чему-либо |
| ▪ to claim to | = | иметь основание требовать, претендовать |
| ▪ to resort to | = | прибегать, обращаться к чему-либо |
| ▪ to test against | = | подтверждать |
| ▪ to keep from | = | воздерживаться от чего-либо |
| ▪ to single out | = | отбирать, выделять |
| ▪ to differ from | = | отличаться от |
| ▪ to liken to | = | уподоблять, сравнивать, приравнивать |
| ▪ to be reserved to | = | быть предназначенным для... |

Ex. 3. Study the Notes below. Then read the texts and translate them into Russian.

NOTES:

- | | | |
|----------------------------|---|--|
| ✓ may <u>very well say</u> | = | <u>вполне</u> можно сказать; |
| by <u>its very nature</u> | = | <u>по самой</u> своей сути (характеру); very перед существительным выполняет функцию прилагательного со значением <u>самый</u> ; |
| ✓ in the sense | = | в смысле; |
| ✓ via [vaie] | = | посредством, через; |
| ✓ regardless of | = | независимо от; |
| ✓ to exemplify | = | пояснять, служить примером; |
| ✓ inevitably | = | неизбежно; |
| ✓ conducive | = | способствующий, благоприятный; |
| ✓ beyond | = | за пределами; |
| beyond dispute | = | бесспорно; |
| ✓ to be at hand | = | быть в распоряжении, рассматриваемый; |
| ✓ insofar as | = | в той степени насколько, насколько; |
| ✓ what not | = | и т. д.; и т. п.; |

- ✓ rather = скорее;
- rather than: = а не – если rather than стоит в середине фразы;
- = вместо того чтобы – если rather than стоит в начале фразы или перед герундием.
- ✓ ‘The former; the latter’ – эти слова-заместители в паре имеют значение “первый (из них)..., последний (из упомянутых)”; во многих случаях в русском предложении следует повторить замененное слово. Однако, они могут использоваться и отдельно друг от друга в значении только первый или только последний из всех, упомянутых ранее, например:

In this paper, we shall take the former approach.

В этой статье мы будем использовать первый метод.

- ✓ Усилительная конструкция ‘do (did) + глагол в инфинитиве без to’: do exist – усилительный глагол ‘do’ используется в утвердительном предложении для усиления значения сказуемого в позиции перед смысловым глаголом, который стоит в форме инфинитива без частицы *to*. Глагол do стоит в нужном времени (does, did), которые при переводе на русский язык переносятся на смысловой глагол с использованием дополнительных слов: действительно, все же, наконец, т. п.

It did cause some difficulties.

Это все же вызвало некоторые трудности.

III. THE NATURE OF MATHEMATICAL ECONOMICS

Mathematical economics is not a distinct branch of economics in the sense that public finance or international trade is. Rather, it is an *approach* to economic analysis, in which the economist makes use of mathematical symbols in the statement of the problem and also draws upon known mathematical theorems to aid in reasoning. It can be micro- or macroeconomic theory, public finance, urban economics, or what not.

Using the term *mathematical economics* in the broadest possible sense, one may very well say that every elementary textbook of economics today exemplifies mathematical economics insofar as geometrical methods are frequently utilized to derive theoretical results. However, mathematical economics is reserved to describe cases employing mathematical techniques beyond simple geometry, such as matrix algebra, differential and integral calculus, differential equations, difference equations, etc.

IV. MATHEMATICAL VERSUS NON-MATHEMATICAL ECONOMICS

Since mathematical economics is merely an approach to economic analysis, it should not and does not differ from the *non* mathematical approach to economic analysis in any fundamental way. The purpose of any theoretical analysis, regardless of the approach, is always to derive a set of conclusions or theorems from a given set of assumptions or postulates via a process of reasoning. The major difference between "mathematical economics" and "literary economics" lies principally in the former, the assumptions and conclusions are stated in mathematical symbols rather than words and in equations rather than sentences. Moreover, in place of literary logic, use is made of mathematical theorems in the reasoning process. It is perhaps beyond dispute that symbols are more conducive to conciseness and preciseness of statement.

Mathematics has the advantage of forcing analysts to make their assumptions explicit at every stage of reasoning. This is because mathematical theorems are usually stated in the "if - then" form.

Granting these points, though, it is necessary to go beyond geometric methods. While geometric analysis has the important advantage of being visual, it also suffers from a serious dimensional limitation. We must resort to the more flexible tool of equations. This reason alone should provide sufficient motivation for the study of mathematical methods beyond geometry.

In short, we see that the mathematical approach has claim to the following advantages: (1) the "language" used is more concise and precise; (2) there exists a wealth of mathematical theorems at our service; (3) it keeps us from the pitfall of an unintentional adoption of unwanted implicit assumptions; and (4) it allows us to treat the general n -variable case.

Against these advantages, one sometimes hears the criticism that a mathematically derived theory is inevitably *unrealistic*. However, this criticism is not valid. Theory is by its very nature an abstraction from the real world. It is a device for singling out only the most essential factors and relationships so that we can study the crux of the problem at hand, free from the many complications that do exist in the actual world. Thus the statement "theory lacks realism" is merely a truism that cannot be accepted as a valid criticism of theory.

In sum, we might liken the mathematical approach to a "mode of transportation" that can take us from a set of postulates (point of departure) to a set of conclusions (destination) at a good speed...

Ex. 4. Brush up your grammar. Find all "-ing" forms in the texts and define them as Participles or Gerunds.

Ex. 5. Find pairs of synonyms:

- | | |
|--------------------|--------------------|
| 1) to resort to | a) to employ |
| 2) frequently | b) elementary |
| 3) thinking | c) to select |
| 4) to help | d) often |
| 5) tool | e) reasoning |
| 6) to comprehend | f) to adopt |
| 7) simple | g) device |
| 8) at our service | h) to understand |
| 9) method | j) to aid |
| 10) to accept | k) technique |
| 11) to single out | l) at our disposal |
| 12) to make use of | m) to turn to |

Ex. 6. Find pairs of opposites:

- | | |
|---------------|-----------------|
| 1) advantage | a) macro |
| 2) imp'licit | b) unrealistic |
| 3) elementary | c) unwanted |
| 4) literary | d) explicit |
| 5) micro | e) disadvantage |
| 6) desirable | f) mathematical |
| 7) realistic | g) fundamental |

Ex.7. Be ready to discuss the following questions in class:

1. Is mathematical economics a distinct branch of economics?
2. What mathematical techniques are used in economic analysis?
3. What is the purpose of any theoretical analysis?
4. How are the assumptions and conclusions stated in mathematical and literary economics?
5. Why are the analysts' assumptions in mathematical expression explicit at every stage of reasoning?
6. What provides sufficient motivation for the study of mathematical methods beyond geometry?
7. Describe the advantages of the mathematical approach in studying economics.
8. Why is the criticism of a mathematically derived theory unrealistic?
9. Is it possible to liken the mathematical approach to a "mode of transportation"?

Ex. 8. Write the following in English:

Математическая экономика предназначена для описания проблем экономики с применением таких математических методов как матричная алгебра, дифференциальное и интегральное исчисление, дифференциальные уравнения, разностные уравнения и т. п.

Математическая экономика как подход к экономическому анализу формирует предположения и выводы в математических символах и уравнениях, так как они более способствуют точности и краткости утверждения.

Одно из преимуществ математического метода анализа экономики это то, что он удерживает нас от принятия непреднамеренных и нежелательных предположений.

Существует критическое замечание, что полученная математическим путем теория неизбежно нереалистична. Но следует отметить, что теория по самой своей сути является абстракцией от реального мира.

Недостатком геометрического анализа является то, что он имеет серьезное мерное ограничение.

Ex. 9. Check your memo:

- | | |
|-------------------------|------------------------|
| ▪ подход | ▪ рассматривать |
| ▪ цель | ▪ нежелательный |
| ▪ методы (2 слова) | ▪ преимущество |
| ▪ формулировка задачи | ▪ независимо от |
| ▪ точность и краткость | ▪ принимать |
| ▪ достаточная мотивация | ▪ обеспечивать |
| ▪ в нашем распоряжении | ▪ инструмент (2 слова) |
| ▪ неизбежно | ▪ рассуждение |
| ▪ ловушка | ▪ допущение |

Ex. 10. Summarize the texts and be ready to speak on the topic: «What is Mathematical Economics?»

UNIT 3

Ex. 1. Practise the pronunciation and give Russian equivalents for the following words without using a dictionary.

- | | | |
|---------------|---------------|---------------|
| ▪ convention | ▪ investment | ▪ visually |
| ▪ combination | ▪ coefficient | ▪ to identify |
| ▪ abstraction | ▪ magnitude | ▪ endogenous |
| ▪ operation | ▪ parameter | ▪ exogenous |
| ▪ application | ▪ an'tithesis | ▪ ingredient |

Ex. 2. Read and translate the words of the same root:

to adopt	–	adopted	–	adoption
to relate	–	related	–	relation
to apply	–	applied	–	application
to accept	–	accepted	–	acceptation
to assume	–	assumed	–	assumption
to contribute	–	contributed	–	contribution
to determine	–	determined	–	determination

Ex. 3. Remember the meaning of the following words and phrases:

- | | | |
|----------------------------------|---|---|
| ✓ in certain ways | = | определенным образом |
| ✓ in the framework of | = | в рамках (пределах) |
| ✓ properly | = | должным образом |
| ✓ inherent | = | присущий, свойственный, существенный |
| ✓ to attach to | = | присоединять, придавать |
| ✓ to stand for | = | обозначать |
| ✓ to refer to | = | ссылаться на |
| ✓ to be referred to as | = | рассматриваться |
| ✓ the assumptions <u>adopted</u> | = | <u>принятые</u> допущения (предположения) |
| ✓ expenditure | = | расход, трата, издержки |
| ✓ profit | = | прибыль, доход |
| ✓ to distinguish from | = | отличать от |
| a distinguished writer | = | выдающийся, известный писатель |
| ✓ in lieu of [lu:] | = | instead of = вместо |
| ✓ in order that | = | с тем, чтобы |

Ex. 4. Read the text and brush up your grammar paying particular attention to the sentences with Modal Verbs. Write out the predicates.

V. INGREDIENTS OF A MATHEMATICAL MODEL

As mentioned before, any economic theory is necessarily an abstraction from the real world. An economic model is merely a theoretical framework, and there is no inherent reason why it must be mathematical. If the model is mathematical, however, it will usually consist of a set of *equations* designed to describe the structure of the model. By relating a number of *variables* to one another in certain ways, these equations give mathematical form to the set of analytical assumptions adopted. Then, through application of the relevant mathematical operations to these equations, we may seek to derive a set of conclusions which logically follow from those assumptions.

Variables, Constants, and Parameters

A *variable* is something whose magnitude can change, i.e., something that can take on different values. Variables frequently used in economics include price, profit, revenue, cost, national income, consumption, investment, imports, exports, and so on. Since each variable can assume various values, it must be represented by a symbol instead of a specific number. For example, we may represent price by P , profit by π , revenue by R , cost by C , national income by Y , and so forth.

Properly constructed, an economic model can be solved to give us the solution values of a certain set of variables, such as the *market-clearing level of price*, or the *profit-maximizing level of output*. Such variables, whose solution values we seek from the model, are known as *endogenous variables* (originating from within). However, the model may also contain variables which are assumed to be determined by forces external to the model, and whose magnitudes are accepted as given data only; such variables are called *exogenous variables* (originating from without). It should be noted that a variable that is endogenous to one model may very well be exogenous to another. In an analysis of the market determination of wheat price (P), for instance, the variable P should definitely be endogenous; but in the framework of a theory of consumer expenditure, P would become instead a datum to the individual consumer, and must therefore be considered exogenous.

Variables frequently appear in combination with fixed numbers or constants, such as in the expressions $7P$ or $0.5R$. A *constant* is a magnitude that does not change and is therefore the antithesis of a variable. When a constant is joined to a variable, it is often referred to as the *coefficient* of that variable. However, a coefficient may be symbolic rather than numerical. We can, for instance, let the symbol a stand for a given constant and use the expression aP in lieu of $7P$ in a model, in order to attain a higher level of generality. To identify its special status, we give it the distinctive name *parametric constant* (or *simple parameter*).

As a matter of convention, parametric constants are normally represented by the symbols a , b , c , or their counterparts in the Greek alphabet: α , β and γ .

But other symbols naturally are also permissible. As for exogenous variables in order that they can be visually distinguished from their endogenous cousins, we shall follow the practice of attaching a subscript 0 to the chosen symbol. For example, if P symbolizes price, then P_0 signifies an exogenously determined price.

Ex. 5. Work in pairs. Discuss the following questions:

1. Can you give the definition to any economic model? What is its difference from a mathematical one?
2. Enumerate the ingredients of a mathematical model known to you.
3. What is the role of equations in the mathematical model?
4. Give the definition to a variable.
5. What variables are frequently used in economics?
6. How can you define endogenous variables?
7. What variables are known as exogenous ones?
8. Why is a constant called the antithesis of a variable?
9. What is the difference between the coefficient and the parameter of a variable?
10. In what way are parametric constants normally represented?

Ex. 6. Find pairs of words in lists A and B that can be used as synonyms.

A	B
to signify	expenditure
for example	frequently
revenue	usually
cost	for instance
value	fixing
often	income
normally	to indicate
determination	magnitude

Ex. 7. Find in the text English equivalents for the words and expressions below:

- | | |
|----------------------|------------------------------|
| ▪ допустимый | ▪ принимать как данное |
| ▪ доход (2 слова) | ▪ рамки |
| ▪ прибыль | ▪ потребитель |
| ▪ потребление | ▪ неясный, нечеткий |
| ▪ величина (2 слова) | ▪ полная противоположность |
| ▪ стоимость | ▪ достигать уровня обобщения |
| ▪ продукция | ▪ обозначать (2 слова) |
| ▪ расходы | ▪ определять (2 слова) |

Ex. 8. Translate the following sentences into English in writing:

Любая экономическая модель непременно является абстракцией от реального мира.

Путем применения соответствующих математических операций к уравнениям мы можем выводить множество заключений, логически следующих из определенных предположений.

Если экономическая модель построена должным образом, то она может быть решена для получения значений определенного множества переменных.

Переменная, которая является эндогенной по отношению к одной модели, например, рыночная цена пшеницы P , вполне может быть экзогенной в рамках теории расходов потребителя.

Переменные часто употребляются в комбинации с фиксированными числами или константами, которые рассматриваются как коэффициенты переменных.

Ex. 9. Summarize the text. Add introductory phrases like:

The text acquaints us with ...

It gives a detailed description of...

The text gives the definitions to ...

Ex. 10. Give the main points of the text in English:

Роль моделей в экономической теории и принятии решений

Экономические модели позволяют выявить особенности функционирования экономического объекта и на основе этого предсказывать будущее поведение объекта при изменении каких-либо параметров. Предсказание будущих изменений, например, повышение обменного курса, ухудшение экономической конъюнктуры, падение прибыли может опираться лишь на интуицию. Однако, при этом могут быть упущены, неправильно определены или неверно оценены важные взаимосвязи экономических показателей, влияющие на рассматриваемую ситуацию. В модели все взаимосвязи переменных могут быть оценены количественно, что позволяет получить более качественный и надежный прогноз.

Для любого экономического субъекта возможность прогнозирования ситуации означает, прежде всего, получение лучших результатов или избежание потерь, в том числе и в государственной политике.

По своему определению любая экономическая модель абстрактна и, следовательно, неполна, поскольку, выделяя наиболее существенные факторы, определяющие закономерности функционирования рассматриваемого экономического объекта, она абстрагируется от других факторов.

UNIT 4

Ex. 1. Read the following international words and guess their meaning:

- | | | |
|--------------|---------------|-----------------|
| ▪ production | ▪ relevance | ▪ technological |
| ▪ function | ▪ equilibrium | ▪ identical |
| ▪ variation | ▪ tax | ▪ total |
| ▪ condition | ▪ identity | ▪ legal |
| ▪ aspect | ▪ to save | ▪ progressively |

Ex. 2. Mind the stress as a determiner of a certain part of speech:

<i>Noun</i>	<i>Verb</i>
ˈincrease	to inc'rease
ˈdecrease	to de'crease
ˈexport	to ex'port
ˈimport	to im'port

Ex. 3. Remember the words and word combinations below:

- | | | |
|----------------------------|----------|--|
| ✓ to distinguish between | = | проводить различие между... |
| ✓ to get up | = | устанавливать |
| ✓ to per'tain | = | иметь отношение к чему-либо |
| ✓ obviously | = | очевидно |
| ✓ to involve | = | включать в себя |
| ✓ total | → cost | = общая стоимость |
| | → income | = суммарный доход |
| | → profit | = суммарная прибыль |
| ✓ as | = | так как, поскольку, <u>по мере того как</u> |
| ✓ pre'requisite | = | предпосылка, (необходимое) предварительное условие |
| ✓ in response to | = | в ответ на |
| ✓ institutional setting | = | установленная структура (рамки) |
| ✓ to behave | = | вести себя |
| ✓ the variable in question | = | данная, обсуждаемая переменная |

NOTE!

The equals sign (=) is read:

"is equal to" or "equals"

The identical equality sign (\equiv) is read:

"is identically equal to"

Ex. 4. Read and translate the following text consulting the notes that follow:

a) Усилительно-выделительная конструкция „it is (was)...that (who, which, when)...” служит для выделения любого члена предложения, кроме сказуемого, обрамляя выделяемое слово – рамочная конструкция. Всю конструкцию следует перевести словами именно, только, как раз, это и дальше продолжать перевод, сохраняя порядок слов английского предложения, например:

It is this last category that is of interest to us.

Именно эта последняя категория представляет для нас интерес.

b) Единичное причастие II в роли определения, стоящее после определяемого слова (постпозитивное причастие), при переводе на русский язык следует ставить перед определяемым словом:

quantity <u>demande</u>	=	<u>требуемое</u> количество/ величина
quantity <u>supplied</u>	=	<u>предоставляемое</u> количество/ величина

Если в предложении рядом стоят два слова с окончанием '-ed', то, как правило, первое и них – определение к слову слева от него, а второе – сказуемое в прошедшем времени, например:

The language used provided to treat the general n-variable case.

Используемый язык позволил рассмотреть общий случай n-переменных.

VI. EQUATIONS AND IDENTITIES

Variables may exist independently, but they do not really become interesting until they are related to one another by equations or by inequalities.

In economic applications we may distinguish between three types of equations: definitional equations, behavioral equations and equilibrium conditions.

A *definitional equation* sets up an identity between two alternate expressions that have exactly the same meaning. For such an equation, the identical-equality sign “=” (read: “is identically equal to”) is often employed in place of the regular equals sign “=”, although the latter is also acceptable. As an example, total profit is defined as the excess of total revenue over total cost; we can therefore write

$$\pi = R - C .$$

A *behavioural equation*, on the other hand, specifies the manner in which a variable behaves in response to changes in other variables. Broadly defined, behavioural equations can be used to describe the general institutional setting of a model, including technological (e.g., production function) and legal (e.g., tax structure) aspects. It is always necessary to adopt definite assumptions

regarding the behaviour pattern of the variable in question. Consider the two cost functions:

$$(2.1) \quad C = 75 + 10Q$$

$$(2.2) \quad C = 110 + Q^2$$

where Q denotes the quantity of output. Since the two equations have different forms, the production condition assumed in each is obviously different from the other. In (2.1), the fixed cost (the value of C when $Q = 0$) is 75, whereas in (2.2) it is 110. The variation in cost is also different. In (2.1), for each unit increase in Q , there is a constant increase of 10 in C . But in (2.2), as Q increases unit after unit, C will increase by progressively larger amounts. Clearly, it is primarily through the specification of the form of the behavioural equations that we give mathematical expression to the assumptions adopted for a model.

The third type of equations, *equilibrium conditions*, have relevance only if our model involves the notion of equilibrium. If so, the equilibrium condition is an equation that describes the prerequisite for the attainment of equilibrium.

Two of the most familiar equilibrium conditions in economics are:

$$Q_d = Q_s \text{ [quantity demanded = quantity supplied]}$$

and

$$S = I \text{ [intended saving = intended investment]}$$

which pertain, respectively, to the equilibrium of a market model and the equilibrium of the national-income model in its simplest form.

Ex. 5. Discuss the following questions with your partner:

1. How many types of equations may we distinguish between in economic applications?
2. What does a definitional equation set up?
3. What does a behavioural equation specify?
4. How is it possible to use behavioural equations in economic analysis?
5. When does an equilibrium conditions equation take place?
6. What are the most familiar equilibrium conditions in economics?
7. To sum up, may variables exist independently?

Ex. 6. Give English equivalents for these words and word combinations:

- | | |
|--------------------------------|---------------------------|
| ▪ соотноситься | ▪ в широком смысле |
| ▪ национальный доход | ▪ рассматривать |
| ▪ структура налога | ▪ очевидно |
| ▪ образец поведения переменной | ▪ определение формы |
| ▪ функция стоимости | ▪ уместность |
| ▪ суммарный доход | ▪ допускать |
| ▪ достижение равновесия | ▪ увеличение / уменьшение |

Ex. 7. Prepare to talk on the topic: «The Ingredients of a Mathematical Model».

Ex. 8. Read the following text and translate it into Russian in writing, minding the number of the nouns below:

<i>Singular</i>	<i>Plural</i>
datum	data
analysis	analyses
hypothesis	hypotheses
antithesis	antitheses

VII. MATHEMATICAL ECONOMICS VERSUS ECONOMETRICS

The term "mathematical economics" is sometimes confused with a related term, "econometrics". As the "metric" part of the latter term implies, econometrics is concerned mainly with the measurement of economic data. Hence it deals with the study of *empirical* observations using statistical methods of estimation and hypothesis testing. Mathematical economics, on the other hand, refers to the application of mathematics to the purely *theoretical* aspects of economic analysis, with little or no concern about such statistical problems as the errors of measurement of the variables under study.

Here, we shall confine ourselves to mathematical economics. That is, we shall concentrate on the application of mathematics to deductive reasoning rather than inductive study, and as a result we shall be dealing primarily with theoretical rather than empirical material.

Indeed, empirical studies and theoretical analyses are often complementary and mutually reinforcing. On the one hand, theories must be tested against empirical data for validity before they can be applied with confidence. In its turn, statistical work needs economic theory as a guide, in order to determine the most relevant and fruitful direction of research.

In one sense, however, mathematical economics may be considered as the more basic of the two: for, to have a meaningful statistical and econometric study, a good theoretical framework — preferably in a mathematical formulation — is indispensable.

NOTES:

✓ estimation	=	оценка
✓ validity	=	обоснованность, правильность (формулы)
✓ concern	=	отношение, касательство
✓ to confine oneself to	=	ограничиваться чем-либо
✓ to reinforce	=	усиливать, укреплять
✓ indispensable	=	совершенно необходимый
✓ mutually	=	взаимно

Ex. 9. Find in the text the words of the same meaning for the following words and phrases:

- | | |
|--------------------------|------------------------|
| ▪ to be concerned with | ▪ to define |
| ▪ mathematical statement | ▪ to control |
| ▪ mistake | ▪ to limit |
| ▪ research | ▪ absolutely necessary |
| ▪ to focus on | ▪ based on observation |

Ex. 10. Summarize the text in English:

Эконометрика

Эконометрика – наука, исследующая количественные закономерности и взаимозависимости в экономике при помощи методов математической статистики. Основа этих методов – корреляционно-регрессионный анализ. Использование современных методов математической статистики началось в биологии. В последней четверти XIX века английский биолог К. Пирсон положил начало современной математической статистике изучением кривых распределения числовых характеристик человеческого организма. Затем он и его школа перешли к изучению корреляций в биологии и построению линейных функций регрессии.

Первые работы по эконометрике появились в конце XIX – начале XX века. В 1897 г. появилась работа одного из основателей математической школы в экономической теории В.Парето, посвященная статистическому изучению доходов населения в разных странах. Была предложена кривая Парето $y = A(x-a)^{-a}$, где x – величина дохода; y – численность лиц, имеющих доход, больший x ; a – минимальный доход; A и a – параметры зависимости, получаемые статистическими методами.

UNIT 5

Ex. 1. Practise the pronunciation of the following words and guess their meaning:

- | | | |
|------------------|-------------|----------------|
| ▪ positive | ▪ category | ▪ component |
| ▪ negative | ▪ fraction | ▪ subcomponent |
| ▪ rational | ▪ scheme | ▪ continuum |
| ▪ irrational | ▪ u'nique | ▪ diameter |
| ▪ non-repeating | ▪ structure | ▪ collectively |
| ▪ characteristic | ▪ pair | ▪ figure |

Ex. 2. Memorize the following:

- | | | |
|-----------------------|---|------------------------------|
| ✓ to lump | = | объединять |
| ✓ circle | = | круг |
| ✓ circumference | = | окружность |
| ✓ 'integer | = | целое число |
| ✓ to fall between | = | размещаться, распределяться |
| ✓ to fill in the gaps | = | заполнять пропуски |
| ✓ designation | = | обозначение |
| ✓ to exhaust | = | исчерпывать |
| ✓ exhaustion | = | истощение |
| ✓ to qualify for | = | определять, быть правомочным |

Ex. 3. Read the text and discuss the questions that follow.

VIII. THE REAL NUMBER SYSTEM

Equations and variables are the essential ingredients of a mathematical model. But since the values that an economic variable takes are usually numerical, a few words should be said about the number system. Here, we shall deal only with so-called "real numbers".

Whole numbers such as 1, 2, 3, ... are called *positive integers*; these are the numbers most frequently used in counting. Their negative counterparts -1, -2, -3, ... are called *negative integers*; these can be employed, for example, to indicate subzero temperatures (in degrees). The number 0 (zero), on the other hand, is neither positive nor negative, and is in that sense unique. Let us lump all the positive and negative integers and the number zero into a single category, referring to them collectively as the *set of all integers*.

Integers, of course, do not exhaust all the possible numbers, for we have fractions, such as $2/3$, $5/4$, and $7/3$, which — if placed on a ruler — would fall between the integers. Also, we have negative fractions, such as $-1/2$ and $-2/5$. Together, these make up the *set of all fractions*.

The common property of all fractional numbers is that each is expressible as a ratio of two integers; thus fractions qualify for the designation *rational numbers* (in this usage, rational means *ratio-nal*). But integers are also rational, because any integer n can be considered as the ratio $n/1$. The set of all integers and the set of all fractions together form the *set of all rational numbers*.

Once the notion of rational numbers is used, however, there naturally arises the concept of *irrational numbers* — numbers that cannot be expressed as ratios of a pair of integers. One example is the number $\sqrt{2} = 1.4142\dots$, which is a non-repeating, non-terminating decimal. Another is the special constant $\pi = 3.1415\dots$ (representing the ratio of the circumference of any circle to its diameter), which is again a non-repeating, non-terminating decimal, as is characteristic of all irrational numbers.

Each irrational number, if placed on a ruler, would fall between two rational numbers, so that, just as the fractions fill in the gaps between the integers on a ruler, the irrational numbers fill in the gaps between rational numbers. The result of this filling-in process is a continuum of numbers, all of which are so-called "real numbers." This continuum constitutes the *set of all real numbers*, which is often denoted by the symbol R . When the set R is displayed on a straight line (an extended ruler), we refer to the line as the *real line*.

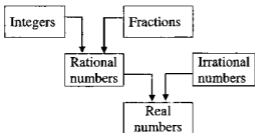


Figure 2.1

In Fig. 2.1 are listed (in the order discussed) all the number sets, arranged in relationship to one another. If we read from bottom to top, however, we find in effect a classificatory scheme in which the set of real numbers is broken down into its component and subcomponent number sets. This figure therefore is a summary of the structure of the real-number system.

In fact, the reason for the term "real" is that there are also "imaginary" numbers, which have to do with the square roots of negative numbers.

Questions:

1. What is the text concerned with?
2. What values does an economic variable take?
3. Does the set of integers include only positive ones?

4. Describe the set of all fractions.
5. What is the common property of all fractional numbers?
6. Can you give the explanation of irrational numbers?
7. What is a continuum of numbers? Why does it constitute the set of all real numbers?
8. What does figure 2.1 illustrate?
9. Why is the real number system so important in mathematical economics?

Ex. 4. Complete the sentences:

1. A number that is greater than zero...
2. A number that is less than zero...
3. All the positive and negative integers and the number zero are referred to as...
4. The set of all integers and the set of all fractions are referred to as...
5. The set of all numbers that cannot be expressed as ratios of a pair of integers...
6. The number representing the ratio of the circumference of any circle to its diameter...
7. The process of filling in the gaps between the rational numbers and the gaps between the integers by fractions is...

Ex. 5. Find pairs of synonyms:

- | | |
|------------------|--------------|
| 1) to constitute | a) to employ |
| 2) to indicate | b) often |
| 3) frequently | c) to form |
| 4) to use | d) notion |
| 5) to display | e) to denote |
| 6) concept | f) to show |

Ex. 6. Give English equivalents for the Russian phrases:

- | | |
|--------------------------------|-----------------------|
| ▪ прямая линия | ▪ десятичная дробь |
| ▪ система действительных чисел | ▪ правильная дробь |
| ▪ процесс заполнения пропусков | ▪ периодическая дробь |
| ▪ температура ниже нуля | ▪ исчерпывать |
| ▪ общее свойство | ▪ обозначение |
| ▪ окружность круга | ▪ в совокупности |
| ▪ множество/подмножество | ▪ линейка |
| ▪ истощение | ▪ часто (2 слова) |

Ex. 7. Get ready to speak on the topic: "The Real Number System".

Ex. 8. Read the following text and translate it into Russian with a dictionary in writing:

IX. THE CONCEPT OF SETS

We have already employed the word "set" several times. Inasmuch as the concept of sets underlies every branch of modern mathematics, it is desirable to familiarize ourselves at least with its more basic aspects.

Set Notation

A set is simply a collection of distinct objects. These objects may be a group of (distinct) numbers, or something else. Thus, all the students enrolled in a particular economics course can be considered a set, just as the three integers 2, 3, and 4 can form a set. The objects in a set are called the *elements* of the set.

There are two alternative ways of writing a set: by *enumeration* and by *description*. If we let S represent the set of three numbers 2, 3, and 4, we can write, by enumeration of the elements,

$$S = \{2, 3, 4\}$$

But if we let I denote the set of all positive integers, enumeration becomes difficult, and we may instead simply describe the elements and write

$$I = \{x \mid x \text{ a positive integer}\}$$

which is read as follows: "I is the set of all (numbers) x, such that x is a positive integer." Note that braces are used to enclose the set in both cases. In the descriptive approach, a vertical bar (or a colon) is always inserted to separate the general symbol for the elements from the description of the elements. As another example, the set of all real numbers greater than 2 but less than 5 (call it J) can be expressed symbolically as

$$J = \{x \mid 2 < x < 5\}.$$

Here, even the descriptive statement is symbolically expressed.

A set with a finite number of elements, exemplified by set S above, is called a *finite set*. Set I and set J, each with an infinite number of elements, are, on the other hand, examples of an *infinite set*. Finite sets are always *denumerable* (set I above), or *non-denumerable* (set J above). In the latter case, there is no way to associate the elements of the set with the natural counting numbers 1, 2, 3, ..., and thus the set is not countable.

Membership in a set is indicated by the symbol \in (a variant of the Greek letter epsilon ϵ for "element"), which is read: "is an element of." Thus, for the two sets S and I defined above, we may write

$$2 \in S \quad 3 \in S \quad 8 \in I \quad 9 \in I \quad (\text{etc.}),$$

but obviously $8 \notin S$ (read: "8 is not an element of set S"). If we use the symbol R to denote the set of all real numbers, then the statement "x is some real number" can be simply expressed by

$$x \in R.$$

Ex. 9. Scan the next text for the answers to the questions below. Then discuss your answers with a partner.

- 1) How many types of relationship between sets may you state?
- 2) Why is the null set unique? What is another term for it?
- 3) In what relationship is neither set a subset of the other?

Relationships between Sets

When two sets are compared with each other, several possible kinds of relationship may be observed. If two sets S_1 and S_2 happen to contain identical elements,

$$S_1 = \{2, 7, a, f\} \quad \text{and} \quad S_2 = \{2, a, 7, f\}$$

then S_1 and S_2 are said to be *equal* ($S_1 = S_2$). Note that the order of appearance of the elements in a set is immaterial. Whenever even one element is different, however, two sets are not equal.

Another kind of relationship is that one set may be a *subset* of another set. If we have two sets

$$S = \{1, 3, 5, 7, 9\} \quad \text{and} \quad T = \{3, 7\}$$

then T is a subset of S , because every element of T is also an element of S . A more formal statement of this is: T is a subset of S if and only if " $x \in T$ " implies " $x \in S$." Using the set inclusion symbols \subset (*is contained in*) and \supset (*includes*), we may then write

$$T \subset S, \text{ or } S \supset T$$

Note that, whereas the \in symbol relates an individual element to a set, the \subset relates a subset to a set. As an application of this idea, we may state on the basis of Fig. 2.1 that the set of all integers is a subset of the set of all rational numbers. Similarly, the set of all rational numbers is a subset of the set of all real numbers.

How many subsets can be formed from the five elements in the set $S = \{1, 3, 5, 7, 9\}$? First of all, each individual element of S can count as a distinct subset of S , such as $\{1\}$, $\{3\}$, etc. The set S itself (with all its five elements) can be considered as one of its own subset. This is, of course, a limiting case, that from which we get the "largest" possible subset of S , namely, S itself.

At the other extreme, the "smallest" possible subset of S is a set that contains no element at all. Such a set is called the *null set*, or *empty set*, denoted by the symbol \emptyset or $\{\}$.

It is extremely important to distinguish the symbol \emptyset or $\{\}$ clearly from the notation $\{0\}$; the former is devoid of elements, but the latter *does* contain an element, zero. The null set is unique; there is only one such set in the whole world, and it is considered a subset of *any* set that can be conceived.

As a third possible type of relationship, two sets may have no elements in common at all. In that case, the two sets are said to be *disjoint*. For example, the set of all positive integers and the set of all negative integers are disjoint sets. A fourth type of relationship occurs when two sets have some elements in common but some elements peculiar to each. In that event, the two sets are neither equal nor disjoint; also, neither set is a subset of the other.

Ex. 10. Summarize the text in English:

Понятие функциональной зависимости

Многочисленные наблюдения и исследования показывают, что в окружающем нас мире величины (например, цена какого-либо товара и величина спроса на этот товар, прибыль фирмы и объем производства этой фирмы, инфляция и безработица и т.п.) существуют не изолированно друг от друга, а напротив, они связаны между собой определенным образом. Понятие функции или функциональной зависимости – одно из основных математических понятий при помощи которых моделируются взаимосвязи между различными величинами, количественные и качественные отношения между различными экономическими характеристиками и показателями.

Понятие функции, как и понятие множества, относится к числу начальных понятий, поэтому оно не определяется, а поясняется. Говорят, что задана *функция* f , если дан закон, согласно которому каждому значению x из некоторого числового множества A ставится в *соответствие* одно вполне определенное значение y из некоторого числового множества B .

Функциональная зависимость между величинами x и y символически обозначается так: $y = f(x)$; говорят, что x – аргумент (независимая переменная), а y – функция (зависимая переменная).

Совокупность всех значений аргумента, каждому из которых соответствует вполне определенное значение функции, называется областью определения функции.

Множество значений, принимаемых y , называется областью изменения функции.

UNIT 6

EQUILIBRIUM ANALYSIS IN ECONOMICS

Ex. 1. Practise the pronunciation of words and give their Russian equivalents:

- | | | |
|---------------|-------------------|-----------------|
| ▪ equilibrium | ▪ to prevail | ▪ to perpetuate |
| ▪ relevance | ▪ to balance | ▪ external |
| ▪ tendency | ▪ to select | ▪ internal |
| ▪ pattern | ▪ to result | ▪ suprapersonal |
| ▪ standard | ▪ to identify | ▪ interaction |
| ▪ analyst | ▪ to characterize | ▪ interrelated |

Ex. 2. Read the text consulting the Notes which follow:

NOTES:

1. Запомните перевод следующих словосочетаний, которые в английском предложении являются **правым определением**, но при переводе ставятся перед определяемым словом

- | | | | |
|---------------------|---|---|-------------|
| a) under study | } | = | изучаемый, |
| under investigation | | | |
| under discussion | | = | обсуждаемый |
| under consideration | } | = | изучаемый, |
| the problem at hand | | | |

- b) under given conditions = при данных условиях

c) перевод постпозитивного Причастия II препозитивно (см. Notes b, Unit 4):

- | | | |
|----------------------------|---|---|
| ▪ variables <u>chosen</u> | = | <u>выбранные</u> переменные |
| ▪ the language <u>used</u> | = | <u>используемый</u> язык |
| ▪ rest <u>involved</u> | = | <u>предполагаемое</u> (подразумеваемое) состояние покоя |

2. Усилительные конструкции:

- | | | |
|---|---|---|
| ▪ <u>It is</u> for this reason <u>that</u> ... | = | <u>именно</u> по этой причине ...
(см. Notes a, Unit 4) |
| ▪ ...factors <u>do</u> actually <u>change</u> ... | = | ...факторы <u>действительно</u> изменяются... (см. Notes, Unit 2) |

3. Mind the following:

- | | | |
|-----------------|---|----------------------------------|
| ✓ thereby | = | таким образом, посредством этого |
| ✓ constellation | = | совокупность, плеяда |
| ✓ to refer to | = | ссылаться на |

✓ to be referred to as	=	называться
✓ to under'score	=	подчеркивать, делать ударение на чем-либо
✓ to obtain	=	to get
✓ 'conscious	=	осознанный

X. THE MEANING OF EQUILIBRIUM

Like any economic term, equilibrium can be defined in various ways. According to one definition, an equilibrium is "a constellation of selected interrelated variables so adjusted to one another that no inherent tendency to change prevails in the model which they constitute." Several words in this definition deserve special attention. First, the word "selected" underscores the fact that there do exist variables which, by the analyst's choice, have not been included in the model. Hence the equilibrium under discussion can have relevance only in the context of the particular set of variables chosen.

Second, the word "interrelated" suggests that, in order for equilibrium to obtain, all variables in the model must simultaneously be in a state of rest. Moreover, the state of rest of each variable must be compatible with that of every other variable; otherwise some variable(s) will be changing, thereby also causing the others to change in a chain reaction, and no equilibrium can be said to exist.

Third, the word "inherent" implies that, in defining an equilibrium, the state of rest involved is based only on the balancing of the internal forces of the model, while the external factors are assumed fixed. Operationally, this means that parameters and exogenous variables are treated as constants.

When the external factors do actually change, there may result a new equilibrium defined on the basis of the new parameter values.

In essence, an equilibrium for a specified model is a situation that is characterized by a lack of tendency to change. It is for this reason that the analysis of equilibrium is referred to as statics. So, an equilibrium is a situation which, if attained, would tend to perpetuate itself, barring any changes in the external forces.

The desirable variety of equilibrium, which we shall refer to as goal equilibrium, will be treated as optimization problems. The non-goal type of equilibrium results not from any conscious aiming at a particular objective but from an impersonal or suprapersonal process of interaction and adjustment of economic forces. Examples of this are the equilibrium attained by a market under given demand and supply conditions and the equilibrium of national income under given conditions of consumption and investment patterns.

In a static-equilibrium model, the standard problem is that of finding the set of values of the endogenous variables which will satisfy the equilibrium condition of the model. This is because once we have identified those values, we have in effect identified the equilibrium state.

Ex. 3. Brush up your grammar. Write out all Passive constructions from the text.

Ex. 4. Give the words with the same meaning to:

- | | |
|--------------------|-----------------|
| ▪ to predominate | ▪ to single out |
| ▪ to determine | ▪ to underline |
| ▪ to force | ▪ to imply |
| ▪ to be considered | ▪ goal |
| ▪ to obtain | ▪ to identify |

Ex. 5. Discuss the questions with your partner:

1. How can equilibrium be defined?
2. What does the word "selected" mean?
3. What does the word "interrelated" mean?
4. Explain the situation when parameters and exogenous factors are treated as constants.
5. What is an equilibrium for a specified model?
6. What is referred to as statics?
7. Can you explain the difference between goal and non-goal types of equilibrium?

Ex. 6. Give a summary of the text. Use introductory phrases like:

The text is concerned with the explanation of...

It gives a detailed (brief) description of...

The author explains ...

In conclusion I'd like to mention that...

Ex. 7. Read the text carefully. Practise asking and answering questions.

XI. PARTIAL MARKET EQUILIBRIUM – A NONLINEAR MODEL

Let the linear demand in the isolated market model be replaced by a quadratic demand function, while the supply function remains linear. Then, if numerical coefficients are employed rather than parameters, a model such as the following may emerge:

$$(3.6) \quad \begin{aligned} Q_d &= Q_s \\ Q_d &= 4 - P^2 \\ Q_s &= 4P - 1 \end{aligned}$$

As previously, this system of three equations can be reduced to a single equation by elimination of variables (by substitution):

$$4 - P^2 = 4P - 1$$

or

$$(3.7) \quad P^2 + 4P - 5 = 0$$

This is a quadratic equation because the left-hand expression is a quadratic function of variable P. The major difference between a quadratic equation and a linear one is that, in general, the former will yield two solution values.

Quadratic Equation versus Quadratic Function

Before discussing the method of solution, a clear distinction should be made between the two terms quadratic equation and quadratic function. According to the earlier discussion, the expression $P^2 + 4P - 5$ constitutes a quadratic function, say, $f(P)$. Hence we may write

$$(3.8) \quad F(P) = P^2 + 4P - 5$$

What (3.8) does is to specify a rule of mapping from P to F (P), such as

P	...	-6	-5	-4	-3	-2	-1	0	1	2	...
f(P)	...	7	0	-5	-8	-9	-8	-5	0	7	...

Although we have listed only nine P values in this table, actually all the P values in the domain of the function are eligible for listing. It is perhaps for this reason that we rarely speak of "solving" the equation $f(P) = P^2 + 4P - 5$, because we normally expect "solution values" to be few in number, but here all P values can get involved. Nevertheless, one may legitimately consider each ordered pair in the table above – such as (-6, 7) and (-5, 0) – as a solution of (3.8), since each such ordered pair indeed satisfies that equation. Inasmuch as an infinite number of such ordered pairs can be written, one for each P value, there is an infinite number of solutions to (3.8). When plotted as a curve, these ordered pairs together yield the parabola in Fig. 3.2.

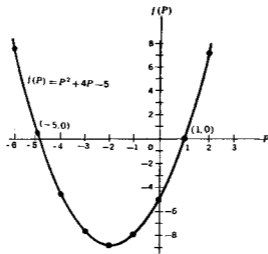


Figure 3.2

In (3.7), where we set the quadratic function $f(P)$ equal to zero, the situation is fundamentally changed. Since the variable $f(P)$ now disappears (having been assigned a zero value), the result is a quadratic equation in the single variable P .* Now that $f(P)$ is restricted to a zero value, only a select number of P values can satisfy (3.7) and qualify as its solution values, namely, those P values at which the parabola in Fig. 3.2 intersects the horizontal axis – on which $f(P)$ is zero. Note that this time the solution values are just P values, not ordered pairs. The solution P values are often referred to as the *roots* of the quadratic equation $f(P) = 0$, or, alternatively, as the *zeros* of the quadratic function $f(P)$.

There are two such intersection points in Fig. 3.2, namely, $(1, 0)$ and $(-5, 0)$. As required, the second element of each of these ordered pairs (the *ordinate* of the corresponding point) shows $f(P) = 0$ in both cases. The first element of each ordered pair (the *abscissa* of the point), on the other hand, gives the solution value of P . Here we get two solutions,

$$\bar{P}_1 = 1 \quad \text{and} \quad \bar{P}_2 = -5$$

but only the first is economically admissible, as negative prices are ruled out.

*The distinctions between quadratic function and quadratic equation just discussed can be extended also to cases of polynomials other than quadratic. Thus, a cubic equation results when a cubic function is set equal to zero.

Ex. 8. Summarize the text in English:

Понятие динамического равновесия в экономике

В экономической теории важным является понятие равновесия, то есть такого состояния объекта, которое он сохраняет при отсутствии внешних воздействий. Задачи экономической динамики включают как описание процессов выхода к состоянию равновесия, так и процессов трансформации самого этого состояния под воздействием внешних сил. Рассмотрим простую экономическую систему в состоянии равновесия и опишем движение такой системы в непрерывном и дискретном случаях. В первом случае динамика системы описывается с помощью дифференциального уравнения, во втором – разностного уравнения.

Дифференциальное уравнение связывает изменения показателя (пусть наша система описывается одним показателем $x(t)$, или просто x) со скоростью его движения x' , или \dot{x} . Будем считать, что скорость изменения показателя x пропорциональна величине его отклонения от равновесного значения x_0 . Иными словами, чем дальше показатель отклонился от равновесного значения, тем быстрее он стремится вернуться к нему. Если в уравнении присутствует только первая производная x по времени, а сама связь линейна, то это линейное дифференциальное уравнение.

UNIT 7

Ex. 1. Practise the pronunciation of the international words which follow and guess their meaning:

- | | | |
|------------------|---------------|---------------|
| ▪ approximation | ▪ linear | ▪ determinant |
| ▪ optimization | ▪ non-linear | ▪ static |
| ▪ transformation | ▪ linearity | ▪ extremely |
| ▪ application | ▪ separate | ▪ logarithm |
| ▪ matrix algebra | ▪ to separate | ▪ concept |

Ex. 2. Memorize the words and phrases given below:

- | | | |
|-------------------------|---|--------------------------------------|
| ✓ to be encountered | = | встречаться, сталкиваться |
| ✓ at the outset | = | вначале |
| ✓ to warrant | = | подтверждать, оправдывать |
| ✓ the solid curve | = | сплошная кривая |
| ✓ ample | = | широкий, достаточный |
| ✓ substantial deviation | = | существенное отклонение |
| ✓ comparative – static | = | сравнительно – статический |
| ✓ applicable | = | применимый, соответствующий |
| ✓ while preserving | = | сохраняя |
| ✓ to proceed to | = | переходить к чему-либо; продолжаться |

XII. LINEAR MODELS AND MATRIX ALGEBRA

Text A

Ex. 3. Read the text carefully and write 7 questions covering the main points. Be ready to discuss them in class.

Matrix algebra can enable us to do many things. In the first place, it provides a compact way of writing an equation system, even an extremely large one. Second, it leads to a way of testing the existence of a solution by evaluation of a *determinant*—a concept closely related to that of a matrix. Third, it gives a method of finding that solution (if it exists). Since equation systems are encountered not only in static analysis but also in comparative-static and dynamic analyses and in optimization problems, you will find ample application of matrix algebra in almost every chapter that is to follow.

However, one slight "catch" should be mentioned at the outset. Matrix algebra is applicable only to linear-equation systems. How realistically linear equations can describe actual economic relationships depends, of course, on the nature of the relationships in question. In many cases, an assumed linear relationship can produce a sufficiently close approximation to an actual

nonlinear relation to warrant its use. In other cases, the closeness of approximation may also be improved by having a separate linear approximation for each segment of a nonlinear relationship, as is illustrated in Fig. 4.1.:

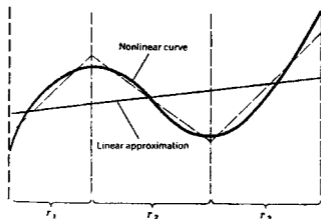


Figure 4.1

If the solid curve is taken as the actual nonlinear relationship, a single linear approximation might take the form of the solid straight line, which shows substantial deviation from the curve at certain points. But if the domain is divided into three regions r_1 , r_2 , and r_3 , we have a much closer linear approximation (broken straight line) in each region.

In yet other cases, while preserving the nonlinearity in the model, we can effect a transformation of variables so as to obtain a linear relation to work with. For example, the nonlinear function

$$y = ax^b$$

can be readily transformed, by taking the logarithm on both sides, into the function

$$\log y = \log a + b \log x$$

which is linear in the two variables ($\log y$) and ($\log x$).

In short, the linearity assumption frequently adopted in economics may in certain cases be quite reasonable and justified. On this note, then, let us proceed to the study of matrix algebra.

Ex. 4. Select the related words from the text for the following and translate them:

close; to apply; to relate; linear; to exist.

Ex. 5. Form adjectives from the given verbs using the suffix “- able” and translate them.

Note, that the suffix “-able” is a highly productive adjective-forming suffix with the meaning “that can or may be done”!

- | | | |
|--------------|--------------|-------------|
| ▪ to apply | ▪ to imagine | ▪ to adopt |
| ▪ to reason | ▪ to rely | ▪ to attain |
| ▪ to predict | ▪ to manage | ▪ to desire |

Ex. 6. Give Russian equivalents for the following:

- | | | |
|-------------|-----------------|----------------------|
| ▪ evolution | ▪ to justify | ▪ in question |
| ▪ curve | ▪ straight line | ▪ ample application |
| ▪ domain | ▪ broken line | ▪ linear application |
| ▪ to obtain | ▪ sufficiently | ▪ to improve |

MATRIX ALGEBRA

Text B

Ex. 1. Read the text consulting the Notes. Translate the first four paragraphs into Russian without a dictionary in writing.

NOTES

- | | | |
|------------------------|---|----------------------------------|
| ✓ provided | = | при условии |
| ✓ prerequisite | = | предпосылка, необходимое условие |
| ✓ 'in'verse | = | противоположность, обратное |
| ✓ to test for | = | исследовать, определить |
| ✓ squareness condition | = | условие квадратности |
| ✓ feasible | = | осуществимый, возможный |
| ✓ to suffice for | = | быть достаточным, удовлетворять |

A linear-equation system, however large, may be written in a compact matrix notation. Furthermore, such an equation system can be solved by finding the inverse of the coefficient matrix, provided the inverse exists. Now we must address ourselves to the questions of how to test for the existence of the inverse and how to find that inverse. Only after we have answered these questions will it be possible to apply matrix algebra meaningfully to economic models.

Conditions for nonsingularity of a matrix

A given coefficient matrix A can have an inverse (i.e., can be “non-singular”) only if it is square. As was pointed out earlier, however, the squareness condition is necessary but not sufficient for the existence of the inverse A^{-1} . A matrix can be square, but singular (without an inverse) nonetheless.

Necessary versus Sufficient Conditions

The concepts of "necessary condition" and "sufficient condition" are used frequently in economics. It is important that we understand their precise meanings before proceeding further.

A necessary condition is in the nature of a prerequisite: suppose that a statement p is true *only if* another statement q is true; then q constitutes a necessary condition of p . Symbolically, we express this as follows:

$$(5.1) p \Rightarrow q$$

which is read: " p only if q ", or alternatively, "if p , then q ". It is also logically correct to interpret (5.1) to mean " p implies q ". It may happen, of course, that we also have $p \Rightarrow w$ at the same time. Then both q and w are necessary conditions for p .

Example 1. If we let p be the statement "a person is a father" and q be the statement "a person is male", then the logical statement $p \Rightarrow q$ applies. A person is a father *only if* he is male, and to be male is a necessary condition for fatherhood. Note, however, that the converse is not true: fatherhood is not a necessary condition for maleness.

A different type of situation is that in which a statement p is true if q is true, but p can also be true when q is not true. In this case, q is said to be a sufficient condition for p . The truth of q suffices for the establishment of the truth of p , but it is not a necessary condition for p . This case is expressed symbolically by

$$(5.2) p \Leftarrow q$$

which is read: " p if q " (without the word "only") – or alternatively, "if q , then p ", as if reading (5.2) backwards. It can also be interpreted to mean " q implies p ".

Example 2. If we let p be the statement "one can get to Europe" and q be the statement "one takes a plane to Europe", then $p \Leftarrow q$. Flying can serve to get one to Europe, but since ocean transportation is also feasible, flying is not a prerequisite. We can write $p \Leftarrow q$, but not $p \Rightarrow q$.

In a third possible situation, q is both necessary and sufficient for p . In such an event, we write

$$(5.3) p \Leftrightarrow q$$

which is read: " p if and only if q " (also written as " p if q "). The double-headed arrow is really a combination of the two types of arrow in (5.1) and (5.2); hence the joint use of the two terms "if" and "only if". Note that (5.3) states not only that p implies q but also that q implies p .

Example 3. If we let p be the statement "there are less than 30 days in the month" and q be the statement "it is the month of February", then $p \Leftrightarrow q$. To have less than 30 days in the month, it is necessary that it be February. Conversely, the specification of February is sufficient to establish that there are less than 30 days in the month. Thus q is a necessary-and-sufficient condition for p .

Ex. 2. Say the following in English. Pay special attention to the Passive Voice.

1. Матричная алгебра может вполне применяться к экономическим моделям.
2. Систему линейных уравнений можно решить путем нахождения обратной матрицы коэффициентов.
3. Понятия «необходимого» и «достаточного» условий часто используются в экономике.
4. Необходимое и достаточное условие выражаются различным образом.
5. Необходимое условие символически читается как "*p only if q*" или, в качестве альтернативы, "*if p than q*".
6. Достаточное условие "*p if q*" (без слова *только*) можно интерпретировать как обратное "*q implies p*".
7. Систему линейных уравнений, какой бы большой она ни была, можно записать компактным обозначением матрицы.

Ex. 3. Write out all the notation readings from the text.

Ex. 4. Find the English equivalents in the above text and learn them:

- | | |
|-------------------|----------------------------|
| ▪ точное значение | ▪ предварительное условие |
| ▪ составлять | ▪ условие квадратности |
| ▪ подразумевать | ▪ существование обратного |
| ▪ устанавливать | ▪ обратное (2 слова) |
| ▪ более того | ▪ в таком случае (2 слова) |
| ▪ тем не менее | ▪ в обратном порядке |

Ex. 5. Summarize the text using phrases like:

The text acquaints us with...

It presents some well-familiar information on the problem of...

I'd like to mention (to remark) that...

UNIT 8

Ex. 1. Think of the meaning of the following words they have in the text given:

- | | | |
|---------------------|-----------------|---------------------------|
| ▪ to concentrate on | ▪ actual | ▪ enlarged |
| ▪ relevance | ▪ initial | ▪ reduced |
| ▪ condition | ▪ numerical | ▪ resulting |
| ▪ application | ▪ stability | ▪ affecting |
| ▪ to ignore | ▪ progressively | ▪ to be associated (with) |

Ex. 2. Read and translate the text consulting the *Essential Vocabulary* below:

- | | | |
|----------------------------|---|--|
| ✓ primary concern | = | первостепенный интерес |
| ✓ ultimately | = | в конечном счете |
| ✓ within a frame | = | в системе, в рамках |
| ✓ to undergo | = | подвергаться, испытывать |
| ✓ in the meantime | = | тем временем, между тем |
| ✓ to envisage | = | предусматривать |
| ✓ to disregard | = | не принимать во внимание, пренебрегать |
| ✓ in response to | = | в ответ на |
| ✓ to fall within the realm | = | находиться в сфере |
| ✓ to pertain to | = | иметь отношение к |
| ✓ propensity | = | склонность, пристрастие |
| ✓ to assign | = | приписывать, задавать (величину) |
| ✓ expenditure | = | затраты, издержки, расходы |

XIII. LIMITATIONS OF STATIC ANALYSIS

In the discussion of static equilibrium in the market or in national income, our primary concern has been to find the equilibrium values of the endogenous variables in the model. A fundamental point that was ignored in such an analysis is the actual process of adjustments and readjustments of the variables ultimately leading to the equilibrium state (if it is at all attainable). We asked only about where we shall arrive, but did not question when or what may happen along the way.

The static type of analysis fails, therefore, to take into account two problems of importance. One is that, since the adjustment process may take a long time to complete, an equilibrium state as determined within a particular frame of static analysis may have lost its relevance before it is even attained, if the exogenous forces in the model have undergone some changes in the meantime. This is the problem of shifts of the equilibrium state. The second is that, even if the adjustment process is allowed to run its course undisturbed, the equilibrium state envisaged in a static analysis may be altogether unattainable. This would be the case of a so-called "unstable equilibrium", which is

characterized by the fact that the adjustment process will drive the variables further away from, rather than progressively closer to, that equilibrium state. To disregard the adjustment process, therefore, is to assume away the problem of attainability of equilibrium.

The shifts of the equilibrium state (in response to exogenous changes) pertain to a type of analysis called *comparative statics*, and the question of attainability and stability of equilibrium falls within the realm of *dynamic analysis*.

Comparative statics, as the name suggests, is concerned with the comparison of different equilibrium states that are associated with different sets of values of parameters and exogenous variables. For purposes of such a comparison, we always start by assuming a given initial equilibrium state.

APPLICATIONS TO COMPARATIVE – STATIC ANALYSIS

National-Income Model

Let us study a slightly enlarged national-income model with three endogenous variables, Y (national income), C (consumption) and T (taxes):

$$Y = C + I_0 + G_0$$

$$(7.17) \quad C = \alpha + \beta(Y - T) \quad (\alpha > 0; 0 < \beta < 1)$$

$$T = \gamma + \delta Y \quad (\gamma > 0; 0 < \delta < 1)$$

The first equation in this system gives the equilibrium condition for national income, while the second and third equations show, respectively, how C and T are determined in the model.

The restrictions on the values of the parameters α , β , γ and δ can be explained thus: α is positive because consumption is positive even if disposable income ($Y - T$) is zero; β is a positive fraction because it represents the marginal propensity to consume; γ is positive because even if Y is zero the government will still have a positive tax revenue (from tax bases other than income); and finally δ is a positive fraction because it represents an income tax rate, and as such it cannot exceed 100 percent. The exogenous variables I_0 (investment) and G_0 (government expenditure) are, of course, nonnegative. All the parameters and exogenous variables are assumed to be independent of one another, so that any of them can be assigned a new value without affecting the others.

This model can be solved for \bar{Y} by substituting the third equation of (7.17) into the second and then substituting the resulting equation into the first. The equilibrium income (in reduced form) is

$$(7.18) \quad \bar{Y} = \alpha - \beta\gamma + I_0 + G_0 / (1 - \beta + \beta\delta)$$

Similar equilibrium values can also be found for the endogenous variables C and T , but we shall concentrate on the *equilibrium income*.

From (7.18), there can be obtained six comparative-static derivatives. Among these, the following three have special policy significance:

$$(7.19) \quad d\bar{Y}/dG_0 = 1/(1 - \beta + \beta \delta) > 0$$

$$(7.20) \quad d\bar{Y}/d\gamma = -\beta/(1 - \beta + \beta \delta) < 0$$

$$(7.21) \quad d\bar{Y}/d\delta = -\beta(\alpha - \beta\gamma + I_0 + G_0)/(1 - \beta + \beta\delta)^2 = -\beta\bar{Y}/(1 - \beta + \beta\delta) < 0$$

The partial derivative in (7.19) gives us the *government-expenditure multiplier*. It has a positive sign here because β is less than 1, and $\beta \delta$ is greater than zero. If numerical values are given for the parameters β and δ , we can also find the numerical value of this multiplier from (7.19). The derivative in (7.20) may be called the *nonincome-tax multiplier*, because it shows how a change in γ , the government revenue from nonincome-tax sources, will affect the *equilibrium income*. This multiplier is negative in the present model because the denominator in (7.20) is positive and the numerator is negative. Lastly, the partial derivative in (7.21) represents an *income-tax-rate multiplier*. For any positive equilibrium income, this multiplier is also negative in the model.

Again, note the difference between the two derivatives $d\bar{Y}/dG_0$ and dY/dG_0 . The former is derived from (7.18), the expression for the equilibrium income. The latter, obtainable from the first equation in (7.17), is $dY/dG_0 = 1$, which is altogether different in magnitude and in concept.

Ex. 3. Give the opposites of the following adjectives by using the prefix “-un” and find their Russian equivalents:

- | | | |
|--------------|------------|--------------|
| ▪ determined | ▪ disputed | ▪ stated |
| ▪ disturbed | ▪ assigned | ▪ important |
| ▪ attainable | ▪ achieved | ▪ disposed |
| ▪ stable | ▪ defined | ▪ obtainable |

Ex. 4. Translate the related words with a dictionary:

- | | | | | |
|-----------------|---|--------------|---|---------------|
| ▪ to attain | – | attainable | – | attainability |
| ▪ to adjust | – | adjustable | – | adjustability |
| ▪ stable | – | unstable | – | stability |
| ▪ to substitute | – | substituting | – | substitution |
| ▪ to determine | – | determined | – | determination |
| ▪ to restrict | – | restricted | – | restriction |
| ▪ significant | – | significance | | |
| ▪ relevant | – | relevance | | |

Ex. 5. Consider the questions below with your partner and then share your thoughts with the entire class:

1. What is the primary concern in discussing static equilibrium in the market or in national income models? What and why is ignored?
2. How can you explain the problem of shifts of the equilibrium state?
3. Why are the exogenous forces in the model so important in the adjustment process?
4. What second problem does the static type of analysis fail to take into account in attaining the equilibrium? Describe the adjustment process in the so-called case of "unstable equilibrium".
5. Why should we always start by assuming a given initial equilibrium state in comparative statics?
6. What model is studied by means of comparative-static analysis in the second section of the text?
7. How can this model be solved for Y (national income) in reduced form?
8. Name the endogenous and exogenous variables of the model.

Ex. 6. Give your translation of the three last paragraphs of the text in writing.

UNIT 9

Ex. 1. Give the meaning of the words and phrases below without consulting a dictionary:

- | | | |
|-----------------------|--------------|-----------------|
| ▪ collective term | ▪ optimality | ▪ connotation |
| ▪ state of technology | ▪ extremum | ▪ to indicate |
| ▪ to function | ▪ criterion | ▪ to constitute |
| ▪ to delineate | ▪ relevant | ▪ alternative |
| ▪ to categorize | ▪ normally | ▪ essence |

Ex. 2. Memorize the following words and word combinations:

- | | | |
|----------------------------|-------------|--|
| ✓ output | = | продукция, выход продукции, производительность |
| ✓ available | = | доступный, имеющийся в наличии |
| ✓ utility | = | полезность, выгодность, практичность |
| ✓ the rate | ↳ of profit | = норма прибыли |
| | ↳ of growth | = норма, темп, скорость, индекс темп роста |
| ✓ matter-of-fact | = | буквальный, беспристрастный |
| ✓ "the quest for the best" | = | поиски наилучшего |
| ✓ connotation | = | дополнительный, подразумеваемый оттенок значения |
| ✓ to accomplish | = | выполнять, добиваться, достигать |

Ex. 3. Read the text and ask as many questions as you can to cover the information given in it:

XIV. OPTIMIZATION PROBLEMS. OPTIMUM VALUES AND EXTREME VALUES

Economics is by and large a science of choice. When an economic project is to be carried out, such as the production of a specified level of output, there are normally a number of alternative ways of accomplishing it. One (or more) of these alternatives will, however, be more desirable than the others from the standpoint of some criterion, and it is the essence of the optimization problem to choose, on the basis of that specific criterion, the best alternative available.

The most common criterion of choice among alternatives in economics is the goal of *maximizing* something (such as maximizing a firm's profit, a consumer's utility, or the rate of growth of a firm or a country's economy) or of *minimizing* something (such as minimizing the cost of producing a given output). Economically, we may categorize such maximization and minimization problems under the general heading of *optimization*, meaning "the quest for the

best". From a purely mathematical point of view, however, the terms "maximum" and "minimum" do not carry with them any connotation of optimality. Therefore, the collective term for maximum and minimum, as mathematical concepts, is the more matter-of-fact designation *extremum*, meaning an extreme value.

In formulating an optimization problem, the first order of business is to delineate an *objective function* in which the dependent variable represents the object of maximization or minimization and in which the set of independent variables indicates the objects whose magnitudes the economic unit in question can pick and choose, with a view to optimizing. We shall therefore refer to the independent variables as *choice variables**. The essence of the optimization process is simply to find the set of values of the choice variables that will yield that desired extremum of the objective function.

For example, a business firm may seek to maximize profit π , that is, to maximize the difference between total revenue R and total cost C . Since, within the framework of a given state of technology and a given market demand for the firm's product, R and C are both functions of the output level Q , it follows that π is also expressible as a function of Q :

$$\pi(Q) = R(Q) - C(Q)$$

This equation constitutes the relevant objective function, with π as the object of maximization and Q as the (only) choice variable. The optimization problem is then that of choosing the level of Q such that π will be a maximum. Note that the optimal level of π is by definition its *maximum* level, but the optimal level of the choice variable Q is itself not required to be either a maximum or a minimum.

Ex. 4. Match the terms from the left column and the definitions from the right column:

- | | |
|-----------------------------|--|
| 1) extremum | a. maximum value |
| 2) the criterion of choice | b. decision values |
| 3) the optimization process | c. to maximize the difference between total revenue R and total cost C |
| 4) optimal level | d. the goal of maximizing something or of minimizing something |
| 5) to maximize profit | e. the object whose magnitude the economic unit in question can pick and choose for optimizing |
| 6) independent variable | f. an extreme value |
| 7) objective function | g. finding the set of values of the choice variables |

Ex. 5. Read the following text and translate it into Russian in writing. Use a dictionary.

XV. NONLINEARITIES IN ECONOMICS

Nonlinearities can arise in various ways. In the production problem in linear programming, the per-unit gross profit of each product was assumed to be a constant. But it can very well be a decreasing function of the output level, either because a larger output tends to depress the market price (average revenue), or because increased production tends to raise the average variable cost of the product. If so, the linear objective function $\pi = c_1x + \dots + c_nx_n$ must be replaced by a nonlinear version, such as $\pi = c_1(x_1)x_1 + \dots + c_n(x_n)x_n$, where $c_j(x_j)$ denotes a decreasing function of the variable x_j .

Similarly, in the constraint section, it may appear that the input requirement for resource i in the production of product j decreases with the output level of product j . For instance, the later units of production may conceivably be processable at greater speed than the earlier ones, so that less machine time will be used up by each successive unit of output. This will, of course, undermine the constancy of the coefficient a_{ij} , as assumed in linear programming. It may also happen that the coefficient a_{ij} depends on the output level, not only of product j , but also of another product k . Then there will arise in the constraint section a term which involves the product of the two variables x_j and x_k , and linearity will again be lost.

Whenever the economic circumstances illustrated above are descriptive of the problem at hand, a nonlinear formulation will be more appropriate than a linear one. Unfortunately, many of the convenient features of linear programming will then become unavailable.

SUPPLEMENTARY READING

I. Demand, Supply, and Prices

Read the text, focus on the following:

1. Economics is a social science.
2. Price and limited resources.
3. Price and company versus customers incentives.

Economics, as we have seen, is concerned with choice in the face of scarcity. This chapter is about prices. Price is defined as what is given in exchange for a good or a service. When the forces of supply and demand are permitted to operate freely, price measures scarcity. As such, prices are a thing of beauty to economists, for they convey critical economic information. When the price of a resource used by a firm is high, the company has a greater incentive to economize on its use. When the price of a good that the firm produces is high, the company has a greater incentive to produce more of that good, and its customers have an incentive to economize on its use. In these ways and others, prices provide our economy with incentives to use scarce resources efficiently.

II. The Role of Prices

Read the text, focus on the following:

1. Scarcity reduces consumption through prices.
2. What determines changes in prices?
3. Economists' versus non-economists' viewpoints on the impersonal forces of supply and demand.

Prices are the way that the participants in the economy communicate with one another. Assume a drought hits the country, reducing drastically the supply of corn. Households will consequently need to reduce their consumption of corn. But how will they know this? Suppose newspapers across the country ran an article informing people that they would have to eat less corn. Would it be read? If so, would people pay attention to it? Why should they? What incentive do they have? How would each family know how much it ought to reduce its consumption? As an alternative to the newspaper, consider the effect of an increase in the price of corn. The higher price quickly and effectively conveys all of the relevant information. Households do not need to know why the price is high. They do not need to know the details of the drought. All they need to know is that there is a greater scarcity of corn, and that they would be wise to

reduce their consumption. The higher price tells them that corn is scarce at the same time that it encourages families to consume less of it.

Prices present interesting problems and puzzles. In the early 1980s, while the price of an average house in Los Angeles went up by 41 percent, the price of a house in Milwaukee, Wisconsin, increased by only 4 percent. Why? During the same period, the price of computers fell dramatically, while the price of bread rose, but at a much slower rate than the price of housing in Los Angeles. Why? The "price" of labor is just the wage or salary that is paid. Why does a physician earn three times as much as a college professor, though the college professor may have performed better in the college courses they took together? Why do women, on average, earn two-thirds the amount that men do? Why did average wage rates fall in the United States between 1973 and 1983? Why is the price of water, without which we cannot live, very low in most cases, but the price of diamonds, which we can surely live without, very high? The simple answer to all of these questions is that in market economies like the United States, price is determined by supply and demand. Changes in prices are determined by changes in supply and demand.

Non-economists see much more in prices than the impersonal forces of supply and demand. One of the events that precipitated the French Revolution was the rise in the price of bread, for which the people blamed the government. More recently, large price changes have given rise to political turmoil in several countries, including Morocco, the Dominican Republic, Russia, and Poland.

On a personal level, individuals tend to blame the owner of an enterprise for the higher prices he charges. It was the landlord who raised the rent on the apartment; it was the oil company or the owner of the gas station who raised the price of gasoline; it was the owner of the movie theater or movie studio who raised the price of movie tickets. These people and companies chose to raise their prices, says the non-economist, in moral indignation. True, replies the economist, but there must be a reason why they decided to raise their prices at this particular time. After all, it is not plausible to say that the landlord or the oil company or the movie theater just had a brainstorm one day and decided to charge more. There must be some factor that made these people and companies believe that a higher price was not a good idea yesterday, but is today.

When all the gas stations in a state or all the landlords in a town start charging roughly the same amount more at the same time, economists argue that there must be a reason. And economists point out that at a different time, these same impersonal forces often oblige these same landlords and oil companies and movie theaters to cut their prices. Economists see prices, then, as symptoms of underlying causes, and they encourage those who are outraged by higher prices to focus on the forces of supply and demand behind the price changes.

III. Demand

Read the text, focus on the following:

1. What is meant by demand?
2. Name variables, other than price, the quantity demanded depends on.
3. Explain, please, why demand curves normally slope downward.

Economists use the concept of demand to describe the quantity of a good or service that a household or firm chooses to buy at a given price. It is important to understand that economists are concerned not just with what people desire, but with what they choose to buy given the spending limits imposed by their budget constraint and given the prices of various goods. Of course, the total demand for a good in the economy depends on more than price. Demand for a product at any price may change with the population (more babies result in a higher demand for diapers) or with the style (demand for miniskirts changes over the years) or with broad social trends (Americans drink more wine and less whiskey now than they did several decades ago).

If analyzing demand meant analyzing all the possible influences on the demand for all possible products, the job of economists would be hopelessly complex. Economists deal with this problem of complexity and multiple factors by focusing on one variable at a time, while keeping all of the other factors fixed. They focus their attention particularly on factors that are most important in causing changes in demand. Of these, the factor that receives the most attention is price. When other changes are important, such as changes in people's income or the structure of the population, then economists take these changes, as well as the effect of these changes on prices, into account.

The Individual Demand Curve

Think about what happens as the price of candy bars changes. At a price of \$5.00, you might never buy one. At \$3.00, you might buy one as a special treat. At \$1.25, you might buy a few, and if somehow the price declined to \$.50, you might buy a lot. Table 4.1 summarizes the weekly demand of one individual, Roger, for candy bars at these different prices. We can see that the lower the price, the larger the quantity demanded.

We can also summarize the information in this table in a graph that shows the quantity Roger demands at each price. By convention, the quantity demanded is measured along the horizontal axis, and the price is measured along the vertical axis. Figure 4.1 plots the points in Table 4.1.

Table 4.1

**ROGER'S DEMAND
FOR CANDY BARS
AT VARIOUS PRICES**

<i>Price</i>	<i>Quantity demanded</i>
\$5.00	0
\$3.00	1
\$2.00	2
\$1.50	3
\$1.25	4
\$1.00	6
\$.75	9
\$.50	15

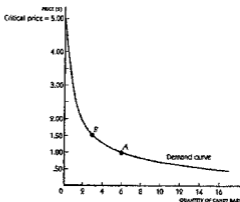
A smooth curve can be drawn to connect the points. This curve is called the demand curve. The demand curve gives the quantity demanded at each price. Thus, if we want to know how many candy bars a week Roger will demand at a price of \$1.00, we simply look along the vertical axis at the price \$1.00, find the corresponding point A along the demand curve, and then read down to the horizontal axis. At a price of \$1.00, Roger buys 6 candy bars each week. Alternatively, if we want to know at what price he will buy just 3 candy bars, we look along the horizontal axis at the quantity 3, find the corresponding point B along the demand curve, and then read across to the vertical axis. Roger will buy 3 candy bars at a price of \$1.50.

The fact that as the price of candy bars increases the quantity demanded decreases can be seen in Table 4.1, or from the fact that the demand curve in Figure 4.1 slopes down already examined the first of these reasons, but the second deserves a closer look. At a high enough price, consumers may drop out of the market for a particular product. In Figure 4.1, for example, Roger exits the market—consumes a quantity of zero—at the price of \$5.00, at which the demand curve hits the vertical axis. Similarly, when prices drop low enough, more consumers will enter the market and thus increase the demand.

Figure 4.1

AN INDIVIDUAL'S DEMAND CURVE

this demand curve shows the quantity of candy bars that Roger consumes at each price. Notice that quantity demanded falls as the price increases, and the demand curve slopes down.



IV. Supply

Read the text, focus on the following:

1. What is meant by supply?
2. Think of variables, other than price, the quantity supplied depends on.
3. Give your explanation why supply curves normally slope upward.

Economists use the concept of supply to describe the quantity of a good or service that a household or firm would like to sell at a particular price. They use the concept to refer to such seemingly disparate choices as the number of

candy bars that firms want to sell and the number of hours that a worker is willing to work. As with demand, the quantity supplied can change according to a variety of factors. A drought can reduce the supply of farm products dramatically. A better production technique may increase the amount supplied of a product. The birth of a child may lead one parent to supply less labor, as that parent takes time off to raise the child, while the other parent may supply more. As in the case of demand, economists focus on price first, while keeping other factors like weather, technology, and so on constant for the moment.

Table 4.3 shows the number of candy bars that the Melt-in-the-Mouth Chocolate Company would like to sell, or supply to the market, at each price. Below \$1.00, the firm finds it unprofitable to produce. At \$2.00, it would like to sell 85,000 candy bars. As the price rises, so does the quantity supplied—at \$5.00, the firm would like to sell 100,000.

Figure 4.4 depicts these points in a graph. The curve drawn by connecting the points is called the **supply curve**. It shows the quantity that Melt-in-the-Mouth will supply at each price, holding all other factors constant. As with the demand curve, we put the price on the vertical axis. The quantity supplied is on the horizontal axis. Thus, we can read point A on the curve as indicating that at a price of \$1.50 the firm would like to supply 70,000 candy bars.

Unlike the demand curve, the typical supply curve slopes upward from left to right; at higher prices, firms will supply more. This is because suppliers find it more profitable to produce the goods with higher prices; the higher prices provide them with an incentive to do so.

Table 4.3

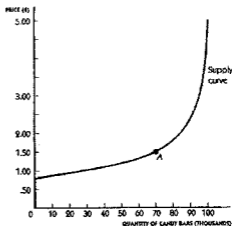
**MELT-IN-THE-MOUTH'S
SUPPLY OF CANDY BARS
AT VARIOUS PRICES**

Price	Supply
\$5.00	100,000
\$3.00	95,000
\$2.00	85,000
\$1.50	70,000
\$1.25	50,000
\$1.00	25,000
\$.75	0
\$.50	0

Figure 4.4

ONE FIRM'S SUPPLY CURVE

The supply curve shows the quantity of a good a firm is willing to produce at each price. Normally a firm is willing to produce more as the price increases, which is why the supply curve slopes upward.



V. Law of Supply and Demand

Read the text, focus on the following:

1. The concept of an economic equilibrium.
2. The equilibrium price occurs at the intersection of the demand and supply curves. Why?
3. Give the definition to the law of supply and demand.

This chapter began with the assertion that supply and demand work together to determine the market price in competitive markets. Figure 4.7 puts a market supply curve and a market demand curve on the same graph to show how this happens. The price actually paid and received in the market will be determined by the intersection of the two curves. This point is labeled E_0 , for equilibrium, and the corresponding price (\$.75) and quantity (20 million) are called, respectively, the equilibrium price and the equilibrium quantity.

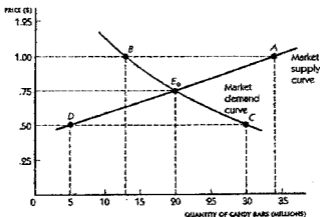


Figure 4.7

Since the term equilibrium will recur throughout the book, it is important to understand the concept clearly. Equilibrium describes a situation where there are no forces (reasons) for change. No one has an incentive to change the result—the price or quantity in the case of supply and demand.

In describing a weight hanging from the end of a spring, physicists also speak of equilibrium. There are two forces working on the weight. Gravity is pulling it down; the spring is pulling it up. When the weight is at rest, it is in equilibrium, with the two forces just offsetting each other. If the weight is pulled down a little bit, the force of the spring will be greater than the force of gravity, and the weight will spring up. In the absence of any further intrusions, the weight will eventually bob back and forth to its equilibrium position.

An economic equilibrium is established in roughly the same way. At the equilibrium price, consumers get precisely the quantity of the good they are willing to buy at that price, and producers sell precisely the quantity they are willing to sell at that price.

Thus, neither producers nor consumers have an incentive to alter the price or quantity. At any other price, there is an incentive for either buyers or sellers to change the price. Consider the price of \$1.00 in Figure 4.7. There is no equilibrium quantity here. First find \$1.00 on the vertical axis. Now look across to find point A on the supply curve, and read down to the horizontal axis; point A tells you that at a price of \$1.00, firms want to supply 34 million candy bars. Now look at point B on the demand curve. Point B shows that at a price of \$1.00, consumers only want to buy 13 million candy bars. Like the weight bobbing on a spring however, this market will work its way back to equilibrium. At a price of \$1.00, there is **excess supply**. As producers discover that they cannot sell as much as they would like at this price, some of them will lower their prices slightly, hoping to take business from other producers. When one producer lowers prices, his competitors will have to respond, for fear that they will end up unable to sell their goods. As prices come down, consumers will also buy more, and so on until the market reaches the equilibrium price and quantity.

Similarly, assume that the price is lower than \$.75, say \$.50. At the lower price, there is **excess demand**: individuals want to buy 30 million candy bars (point C), while firms only want to produce 5 million (point D). Consumers unable to purchase all they want will offer to pay a little bit more; other consumers, afraid of having to do without, will match these higher bids or raise them. As prices start to increase, suppliers will also have a greater incentive to produce more. Again the market will tend toward the equilibrium point.

At equilibrium, no purchaser and no supplier has an incentive to change the price or quantity. The observation that in competitive market economies actual prices tend to be the equilibrium prices, at which demand equals supply, is called the **law of supply and demand**. It is important to note that this law does not mean that at every moment of time the price is precisely at the intersection of the demand and supply curves. As with the example of the weight and the spring, the market may bounce around a little bit when it is in the process of adjusting. The law of supply and demand does say that when a market is out of equilibrium, there are predictable forces for change.

VI. CLOSE-UP: THE STRUCTURE OF ECONOMIC MODELS

Read the text without a dictionary and check your memory: comment on the key point of each paragraph. Work in groups of three.

Every economic model, including the model of how supply and demand determine the equilibrium price and quantity in a market, is constructed of three kinds of relationships: identities, behavioral relationships, and equilibrium relationships. Recognizing these component parts will help in understanding how economists think and understanding the source of many of their disagreements.

As described in the text, the demand curve represents a relationship between the price and the quantity demanded. The statement that normally, as prices rise, the quantity of a good demanded decreases is a description of how individuals behave. It is called a behavioral relationship. The supply curve for each firm is also a behavioral relationship.

Economists disagree over behavioral relationships, in at least two ways. First, they may differ over the strength of the connection. For any given product, does a change in price lead to a large change in the quantity demanded or a small one? Second, economists may sometimes even disagree over the direction of the effect. As later chapters will discuss, there are some special cases where a higher price may actually lead to a lower quantity supplied.

The statement that the market demand is equal to the sum of the individual demands is an identity. An identity is a statement that is true according to the definition of the terms; in other words, market demand is defined to be the sum of the demands of all individuals. Similarly, it is an identity that market supply is equal to the sum of the supplies of all firms; the terms are defined in that way. Economists rarely disagree over identities, since disagreements over definitions are pointless.

Finally, an equilibrium relationship exists when there are no forces for change. In the supply and demand model, the equilibrium occurs when the quantity demanded is equal to the quantity supplied. An equilibrium relationship is not the same as an identity, it is possible for the economy to be out of equilibrium, at least for a time. Of course, being out of equilibrium implies that there are forces for change pushing toward equilibrium. But an identity must always hold true at all times, as a matter of definition.

Economists usually agree about what an equilibrium would look like, but they often differ on whether the forces pushing the markets toward equilibrium are strong or weak, and thus on whether the economy is usually fairly close to equilibrium or sometimes rather far from it.

VII. Read the text carefully and discuss the following:

1. The three sources of disagreement among economists.
2. The consequences of the policy taken.
3. Disagreements about the models of the economy are productive.

WHY ECONOMISTS DISAGREE

Economists are frequently called upon to make judgments on matters of public policy.

Should the government reduce the deficit? Should inflation be reduced? If so, how?

In these public policy discussions, the disagreements among economists often get considerable attention. But disagreement can be a productive way of learning more, if approached properly. Economists try carefully to define the sources and reasons for their differences.

Two major sources of disagreement exist within the scientific realm of economics. First, economists may differ over what is the appropriate model of the economy. They may disagree about how well people and firms are able to perceive and calculate their self-interest, and whether their interactions take place in a competitive or a noncompetitive market. Different models will often produce different results. Often the data that we currently have available do not allow us to say which of two competing models provides a better description of some market.

Second, even when they agree about the appropriate theoretical model, economists may disagree about quantitative magnitudes, and accordingly their predictions will differ. They may agree, for instance, that reducing the tax on interest income will encourage individuals to save more. But some economists may claim, on the basis of their studies, that individuals will increase their savings only a little, others, that people will increase their savings a lot. Again, many of these disagreements arise because of the lack of relevant data. We may have considerable data concerning savings in the United States over the past century. But institutions and economic conditions today are markedly different from those of fifty or even ten years ago.

There is another source of disagreement, but this one lies outside the scientific realm. It is common for economists to be asked questions like "Should the government cut the capital gains tax to encourage savings?" "Should the government cut taxes to stimulate the economy and reduce unemployment?" To answer such questions, economists must determine the consequences of the policy in question, which makes it necessary first to formulate a model of the economy or the market. Even if the goals of the policy are clear, disagreements may occur for either of the reasons given above. But if

the goals are unclear, then the economist's own values may intrude, and this is where the third source of disagreement comes in.

There are generally many consequences of any policy, some beneficial, some harmful. In comparing two policies, one may benefit some people more, another may benefit others. One policy is not unambiguously better than another. It depends on what you care more about. A cut in the tax on the profits from the sale of stocks might encourage savings, but at the same time, most of the benefits accrue to the very wealthy; hence, it increases inequality. A reduction in taxes to stimulate the economy may reduce unemployment, but it may also increase inflation. Even though two economists agree about the model, they may make different recommendations.

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THE GREEK ALPHABET

Α	α	alpha
Β	β	beta
Γ	γ	gamma
Δ	δ	delta
Ε	ε	epsilon
Ζ	ζ	zeta
Η	η	eta
Θ	θ	theta
Ι	ι	iota
Κ	κ	kappa
Λ	λ	lambda
Μ	μ	mu
Ν	ν	nu
Ξ	ξ	xi
Ο	ο	omicron
Π	π	Pi
Ρ	ρ	rho
Σ	σ	sigma
Τ	τ	tau
Υ	υ	upsilon
Φ	φ (or ϕ)	phi
Χ	χ	chi
Ψ	ψ	psi
Ω	ω	omega

MATHEMATICAL SYMBOLS

1. Sets

$a \in S$	a is an element of (belongs to) set S
$b \notin S$	b is not an element of set S
$S \subset T$	set S is a subset of (is contained in) set T
$T \supset S$	set T includes set S
$A \cup B$	the union of set A and set B
$A \cap B$	the intersection of set A and set B
\bar{S}	the complement of set S
$\{ \}$ or \emptyset	the null set (empty set)
$\{a, b, c\}$	the set with elements a , b , and c
$\{x \mid x \text{ has property } P\}$	the set of all objects with property P
$\min \{a, b, c\}$	the smallest element of the specified set
\mathbb{R}	the set of all real numbers
\mathbb{R}^2	the two-dimensional real space
\mathbb{R}^n	the n -dimensional real space
(x, y)	ordered pair
(x, y, z)	ordered triple
(a, b)	open interval from a to b
$[a, b]$	closed interval from a to b

2. Matrices and Determinants

A' or A^t	the transpose of matrix A
A^{-1}	the inverse of matrix A
$ A $	the determinant of matrix A
$ J $	Jacobian determinant
$ H $	Hessian determinant
$ H $	bordered Hessian determinant
$r(A)$	the rank of matrix A
O	null matrix (zero matrix)
$u \cdot v$	the inner product (dot product) of vectors u and v
$u \cdot v$	the scalar product of two vectors

3. Calculus

Given $y=f(x)$, a function of a single variable x :

$\lim_{x \rightarrow 0} f(x)$	the limit of $f(x)$ as x approaches infinity
Dx	differential of x
dy	the first differential of y
d^2y	the second differential of y
$\frac{dy}{dx}$ or $f'(x)$	the first derivative of the function $y = f(x)$
$\frac{dy}{dx} \Big _{x=x_0}$ or $f'(x_0)$	the first derivative evaluated at $x = x_0$
$f(x)$	function of x
$\frac{d^2y}{dx^2}$ or $f''(x)$	the second derivative of the function $y = f(x)$
$\frac{d^n y}{dx^n}$ or $f^{(n)}(x)$	the n th derivative of $y = f(x)$
$\left\{ \begin{array}{l} f'(x) \\ \infty \end{array} \right.$	function primed
	infinity
$\int f(x)dx$	indefinite integral of $f(x)$
$\int_a^b f(x)dx$	definite integral of $f(x)$ from $x=a$ to $x=b$
dy/dx	derivative of y with respect to x

Given the function $y=f(x_1, x_2, \dots, x_n)$:

$\frac{\partial y}{\partial x_i}$ or f_i	the partial derivative of f with respect to x_i
$\frac{dy}{dx_i}$	the total derivative of f with respect to x_i
$\frac{\delta y}{\delta x_i}$	the partial derivative of f with respect to x_i
$\Delta x \rightarrow 0$	delta x tends to zero

4. Differential and Difference Equations

$y \equiv \frac{dy}{dt}$	the time derivative of y
Δy_t	the first difference of y_t
$\Delta^2 y_t$	the second difference of y_t
y_p	particular integral
y_c	complementary function

5. Others

$\sum_{i=1}^n x_i$	the sum of x_i as i ranges from 1 to n (or summation)
$p \Rightarrow q$	p only if q (p implies q)
$p \Leftarrow q$	p if q (p is implied by q)
$p \Leftrightarrow q$	p if and only if q
iff	if and only if
$ m $	the absolute value of the number m
$n!$	n factorial $\equiv n(n-1)(n-2)\dots(3)(2)(1)$
$\log_b x$	the logarithm of x to base b
$\log_e x$ or $\ln x$	the natural logarithm of x (to base e)
e	the base of natural logarithms and natural exponential functions
$\sin \theta$	sine function of θ
$\cos \theta$	cosine function of θ
R_n	the remainder term when the Taylor series involves an n th-degree polynomial
$\log_{10} 3$	logarithm of 3 to the base of 10
A_n	"a sub n "
e^π	"e to the power π "
()	parenthesis
{ }	braces
$\frac{1}{2}$	one second
$-\frac{7}{5}$	minus seven fifth
$2 : 50 = 4 : x$	two is to fifty as four to x